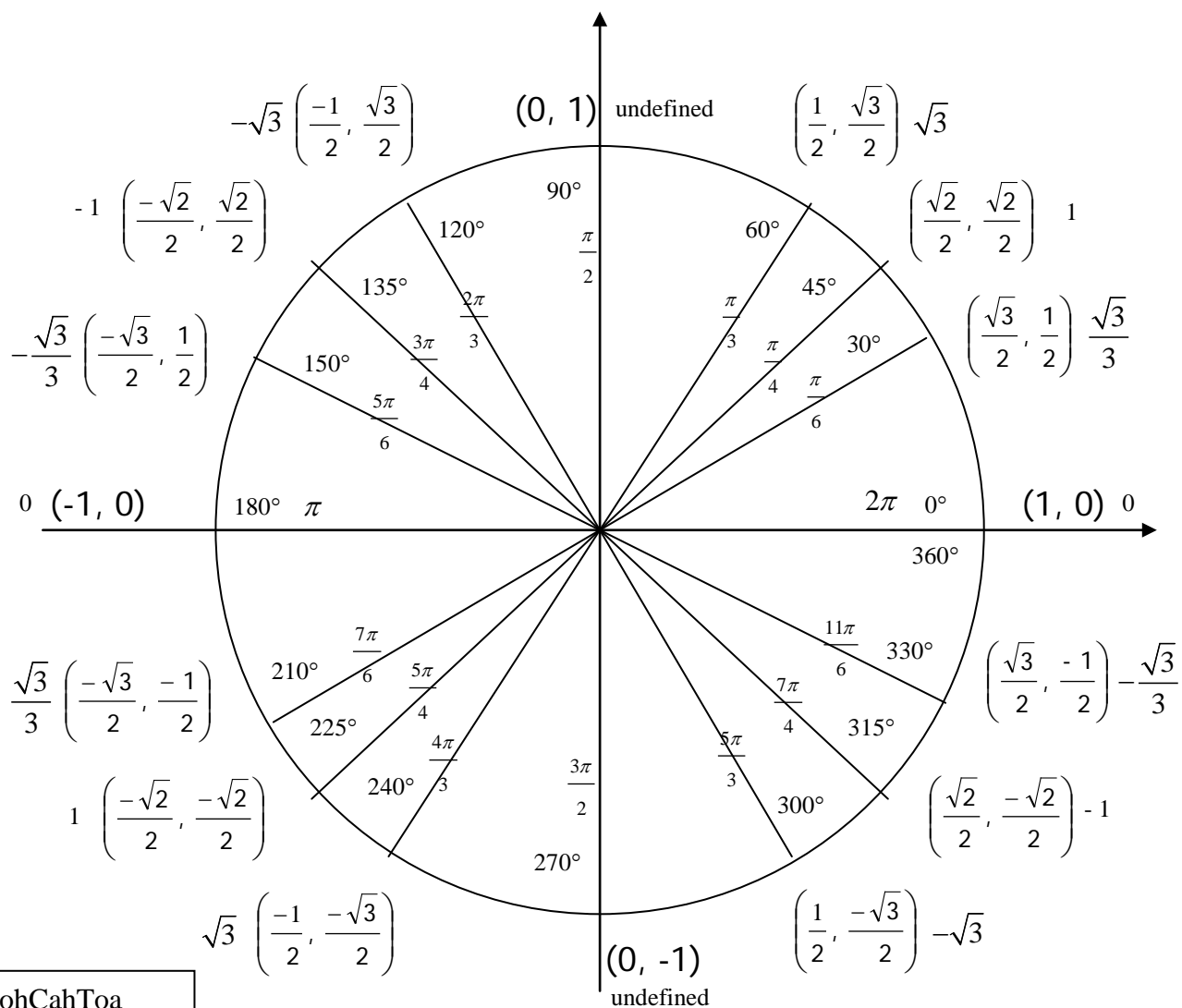


# UNIT CIRCLE



SohCahToa

**Pythagorean Triples:**  
 3, 4, 5  
 5, 12, 13  
 8, 15, 17  
 7, 24, 25

**Equation of a circle:**  
 $x^2 + y^2 = r^2$

**Law of Sines:**  

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

**Law of Cosines:**  
 $c^2 = a^2 + b^2 - 2ab \cos C$   
 $a^2 = b^2 + c^2 - 2bc \cos A$   
 $b^2 = a^2 + c^2 - 2ac \cos B$

**Reciprocals:**  
 $1 \leftrightarrow 1$   $und \leftrightarrow 0$   
 $2 \leftrightarrow \frac{1}{2}$   $\frac{\sqrt{3}}{2} \leftrightarrow \frac{2\sqrt{3}}{3}$   
 $\frac{\sqrt{2}}{2} \leftrightarrow \sqrt{2}$   $\frac{\sqrt{3}}{3} \leftrightarrow \sqrt{3}$

**Trig Functions:**  
 $\sin \theta = \frac{y}{r}$   $\csc \theta = \frac{r}{y}$   
 $\cos \theta = \frac{x}{r}$   $\sec \theta = \frac{r}{x}$   
 $\tan \theta = \frac{y}{x}$   $\cot \theta = \frac{x}{y}$

# TRIGONOMETRIC IDENTITIES

## Pythagorean Relationships

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$1 - \sin^2 \theta = \cos^2 \theta$$

$$1 - \cos^2 \theta = \sin^2 \theta$$

$$1 + \tan^2 \theta = \sec^2 \theta$$

$$\sec^2 \theta - 1 = \tan^2 \theta$$

$$\sec^2 \theta - \tan^2 \theta = 1$$

$$1 + \cot^2 \theta = \csc^2 \theta$$

$$\csc^2 \theta - 1 = \cot^2 \theta$$

$$\csc^2 \theta - \cot^2 \theta = 1$$

## Power-Reducing Identities

$$\sin^2 x = \frac{1 - \cos 2x}{2}$$

$$\cos^2 x = \frac{1 + \cos 2x}{2}$$

$$\tan^2 x = \frac{1 - \cos 2x}{1 + \cos 2x}$$

## Odd/Even

$$\sin(-\theta) = -\sin \theta$$

$$\cos(-\theta) = \cos \theta$$

$$\csc(-\theta) = -\csc \theta$$

$$\sec(-\theta) = \sec \theta$$

$$\tan(-\theta) = -\tan \theta$$

$$\cot(-\theta) = -\cot \theta$$

## Sum/Difference

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

$$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$$

$$\tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta}$$

## Cofunction

$$\sin \theta = \cos(90^\circ - \theta)$$

$$\cos \theta = \sin(90^\circ - \theta)$$

$$\tan \theta = \cot(90^\circ - \theta)$$

$$\cot \theta = \tan(90^\circ - \theta)$$

$$\sec \theta = \csc(90^\circ - \theta)$$

$$\csc \theta = \sec(90^\circ - \theta)$$

## Reciprocal/Quotient Relationships

$$\csc \theta = \frac{1}{\sin \theta} \quad \sin \theta = \frac{1}{\csc \theta}$$

$$\sec \theta = \frac{1}{\cos \theta} \quad \cos \theta = \frac{1}{\sec \theta}$$

$$\cot \theta = \frac{1}{\tan \theta} \quad \tan \theta = \frac{1}{\cot \theta}$$

$$\cot \theta = \frac{\cos \theta}{\sin \theta} \quad \tan \theta = \frac{\sin \theta}{\cos \theta}$$

## Double Angle Identities

$$\sin 2\theta = 2 \sin \theta \cdot \cos \theta \quad \tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

$$\cos 2\theta = \cos^2 \theta - \sin^2 \theta$$

$$\cos 2\theta = 1 - 2 \sin^2 \theta$$

$$\cos 2\theta = 2 \cos^2 \theta - 1$$

## Half Angle Identities

$$\sin \frac{\alpha}{2} = \pm \sqrt{\frac{1 - \cos \alpha}{2}} \quad \tan \frac{\alpha}{2} = \pm \sqrt{\frac{1 - \cos \alpha}{1 + \cos \alpha}}$$

$$\cos \frac{\alpha}{2} = \pm \sqrt{\frac{1 + \cos \alpha}{2}} \quad \tan \frac{\alpha}{2} = \frac{1 - \cos \alpha}{\sin \alpha} = \frac{\sin \alpha}{1 + \cos \alpha}$$

Whether the + or - sign is determined by the quadrant of angle  $\frac{\alpha}{2}$ , denominator  $\neq 0$