BC Calculus
Derivatives – Chain, Trig.

Name: key

Find the derivative:

1. $f(x)=8\sqrt{x^4-4x^2}$

$=8(x^4-4x^2)^{\frac{1}{2}}$

$f'(x)=8\left(\frac{1}{2}\right)(x^4-4x^2)^{-\frac{1}{2}}(4x^3-8x)$

$\frac{4(4x^3-8x)}{\sqrt{(x^4-4x^2)}}$

2. $f(x)=\sqrt[4]{\frac{2x-5}{5x+2}}$

$f'(x)=\frac{1}{4}\left(\frac{2x-5}{5x+2}\right)^{-\frac{3}{4}}\left(\frac{(5x+2)(2)-(2x-5)(5)}{(5x+2)^2}\right)$ (10x+4, −10x+25)

$=\frac{1}{4}\left(\frac{2x-5}{5x+2}\right)^{-\frac{3}{4}}\left(\frac{29}{(5x+2)^2}\right)$

3. $f(x)=\left(x+\frac{1}{x}\right)\left(x^2-\frac{1}{x^2}\right)$

$\left(\frac{x^2+1}{x}\right)\left(\frac{x^4-1}{x^2}\right)=\frac{x^3((x^2+1)(4x)+(x^4-1)(2x))-3x^2((x^2+1)(x^4-1))}{x^6}$

$f'(x)=\left(x+\frac{1}{x}\right)(2x+2x^{-3})+\left(x^2-\frac{1}{x^2}\right)(1+-1x^{-2})$

$\frac{4x^4(x^2+1)+2x^4(x^4-1)-3x^4(\ )}{x^6}$

$\frac{4x^2(x^2+1)+2x^2(x^4-1)-3(\ )}{x^4}$

4. $f(x)=\frac{x^6+4x^3+6}{(x^4-2)^2}$

$\frac{(x^4-2)^2(6x^5+12x^2)-(x^6+4x^3+6)(2(x^4-2)(4x^3))}{(x^4-2)^4}$

5. $f(x)=\frac{1+u}{1+u^2}\quad u=x^2-1$

$\frac{(x^4-2x^2+2)(2x)-(x^2)(4x^3-4x)}{(x^4-2x^2+2)^2}=\frac{-2x^5+4x}{(x^4+2x^2+2)^2}$

6. $f(x)=2\sin 3x\cos 4x$

$2\sin 3x(-\sin 4x)+\cos 4x(2\cos 3x)\cdot 3$

$-8\sin 3x\sin 4x+6\cos 4x\cos 3x$

7. $r(\theta) = \cos(1+\sin\theta)$

$r' = -\sin(1+\sin\theta)\cos\theta$

8. $y = \left(1+\cot\left(\frac{2}{x}\right)\right)^{-2}$

$y' = -2\left(1+\cot\left(\frac{2}{x}\right)\right)^{-3}\left(-\csc^2\left(\frac{2}{x}\right)\right)\left(-2x^{-2}\right)$

$= \dfrac{-4\csc^2\left(\frac{2}{x}\right)}{x^2\left(1+\cot\left(\frac{2}{x}\right)\right)^3}$

9. $y = \sqrt{\sin 3x}$

$y' = \frac{1}{2}(\sin 3x)^{-\frac{1}{2}}(\cos 3x)\,3 = \frac{3}{2}(\cos 3x)(\sin 3x)^{-\frac{1}{2}}$

10. $y = \cos^{-1}\left(\frac{1}{x}\right)$

$y' = \dfrac{-1}{\sqrt{1-\left(\frac{1}{x}\right)^2}}\left(-x^{-2}\right) = \dfrac{1}{x^2\sqrt{\frac{1}{x^2}-\frac{1}{x^2}}} = \dfrac{1}{x\sqrt{x^2-1}}$

11. $y = \dfrac{1}{\sin^{-1}(2x)}$

$y' = \dfrac{-1\left(\frac{1}{\sqrt{1-(2x)^2}}\cdot 2\right)}{\left(\sin^{-1}(2x)\right)^2} = \dfrac{-2}{\sqrt{1-4x^2}\left(\sin^{-1}(2x)\right)^2}$

12. $y = \tan^{-1}\left(t^2\right)$

$y' = \dfrac{1}{1+(t^2)^2}\cdot 2t = \dfrac{2t}{1+t^4}$