Calculus

Name: Key

Chapter 6 Review

1. $\int \frac{3}{x^2-4x-5}dx$

$\frac{3}{(x-5)(x+1)} = \frac{A}{x-5} + \frac{B}{x+1}$

$3 = A(x+1) + B(x-5)$

$3 = -6B$

$B = -\frac{1}{2}$

$A = \frac{1}{2}$

$\frac{1}{2}\ln|x-5| - \frac{1}{2}\ln|x+1| + C$

2. $\int e^{2x}\cos x\,dx$

$= \frac{e^{2x}\sin x}{5} + \frac{2}{5}e^{2x}\cos x + C$

3. $\cancel{\int 8\cos^4 x\,dx}$

4. $\int \cos^3 x\,dx = \sin x - \frac{\sin^3 x}{3} + C$

$\cos x(1-\sin^2 x)dx$

$u = \sin x \quad du = \cos x\,dx$

5. $\int \ln x\,dx = x\ln x - x + C$

6. $\int x\sec^2 x\,dx \quad x\tan x + \ln|\cos x| + C$

7. $\int \frac{3x}{4x^2+1}dx \quad \frac{3}{8}\ln|4x^2+1| + C$

$u = 4x^2+1$

$du = 8x\,dx$

8. $\int x^3 \ln x\,dx = \frac{x^4}{4}\ln x - \frac{x^4}{16} + C$

9. $\int \frac{3x^2}{\sqrt{x^3+1}}dx = \int u^{-\frac{1}{2}}du$

$u = x^3+1$

$du = 3x^2dx$

$= 2(x^3+1)^{\frac{1}{2}} + C$

Solve the initial value problems:

10. $\frac{dx}{dt} = .001x(100-x)$ with $x(0) = 10$

$x = \frac{100}{1+9e^{-(.001)(100)t}} = \frac{100}{1+9e^{-.1t}}$

11. $\frac{dy}{dx} = yx$ and y = 2 when x = 0

$\frac{dy}{y} = x\,dx$

$\ln|y| = \frac{x^2}{2} + C$

$Ce^{\frac{x^2}{2}} = y \quad 2e^{\frac{x^2}{2}} = y$

12. $\frac{dy}{dx} = \frac{-x}{y+1}$ and y = -3 when x = 0

$(y+1)dy = -x\,dx$

$\frac{y^2}{2} + y = -\frac{x^2}{2} + C$

$\frac{y^2}{2} + y = -\frac{x^2}{2} + \frac{3}{2}$

13. $\int 5^x dx = \frac{5^x}{\ln 5} + C$

14. Use Euler's method:

$\frac{dy}{dx} = 2x + y$ if *f(0)* = 0 find *f(1)* using *dx* = .2

(0, 0)

(.2, 0)

(.4, .08)

(.6, .256)

(.8,

(1,

15. $\int \frac{4x+41}{x^2+3x-10}dx =$

$\boxed{.97764}$

$4x+41 = \frac{A}{x-2} + \frac{B}{x+5}$

$4x+41 = A(x+5) + B(x-2)$

$4x+41 = (A+B)x + 5A - 2B$

$A+B = 4$

$5A-2B = 41$

1. 1974 AB 7 **No Calculator**
The rate of change in the number of bacteria in a culture is proportional to the number present. In a certain laboratory experiment, a culture had 10,000 bacteria initially, 20,000 bacteria at time t1 minutes, and 100,000 bacteria at (t1 + 10) minutes.

(a) In terms of t only, find the number of bacteria in the culture at any time t minutes, $t > 0$.

(b) How many bacteria were there after 20 minutes?

(c) How many minutes had elapsed when the 20,000 bacteria were observed?

$P = \dfrac{M}{1 + Ae^{-kMt}}$

$A = \dfrac{M - P_0}{P_0}$

2. The rate at which a rumor spreads through a high school of 2000 students can be modeled by the differential equation $\dfrac{dP}{dt} = 0.003P(2000 - P)$, where P is the number of students who have heard the rumor t hours after 9AM.

1000

(a) How many students have heard the rumor when it is spreading the fastest? Justify your answer.
(b) If $P(0) = 5$, solve for P as a function of t. $P = \dfrac{2000}{1 + 399e^{-6t}}$ $\dfrac{2000-5}{5}$
(c) Use your answer to (b) and your graphing calculator to determine how many hours have passed when half the student body has heard the rumor.

$t = .998$

3. Given the differential equation $\dfrac{dy}{dx} = x + y$. Let $y = f(x)$ be the particular solution to the given differential equation with the initial condition $f(4) = -1$. Use Euler's method, starting at $x = 4$ with two steps of equal size, to approximate $f(4.2)$. Show the work that leads to your answer.

$(4, -1)$ $y_n = -1 + .1(3)$
$(4.1, -.7)$ $-.70$
$(4.2, -.36)$ $y_n = -.7 + .1(3.4)$ $.34$
$.36$

4. The population $P(t)$ of a species satisfies the logistic differential equation $\dfrac{dP}{dt} = P\left(2 - \dfrac{P}{5000}\right)$, where the initial population is $P(0) = 3000$ and t is the time in years. What is $\lim_{t\to\infty} P(t)$?

(A) 2500 (B) 3000 (C) 4200 (D) 5000 (E) 10,000

$\dfrac{1}{5000}P(10,000 - P)$