10.1 Parametric Equations #235

equations that describe motion in terms of time because they have an additional variable (the parameter) - where an item is at a particular moment in time

there are 2 parts to a parametric equation - in terms of time

$x = f(t)$ this is one set of equations

$y = g(t)$

t is the parameter

plot points as (x, y)

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Make a table of values, sketch the curve, indicate direction. Then eliminate the parameter.

$$x = \sqrt{t+1}$$

$$y = t + 2$$

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Make a table of values, sketch the curve, indicate direction. Then eliminate the parameter.

$$x = t^2 - 3$$
$$y = 2t$$
$$-2 \le t \le 3$$

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Eliminate the parameter in the following equation:

$$x = 3\cos t$$
$$y = 4\sin t$$

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Parametric Equations and Calculus

If a smooth curve is represented by parametric equations, then the slope of the curve at a point (x, y) is given by:

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} \qquad \text{where} \quad \frac{dx}{dt} \neq 0$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}}$$

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Ex. 1 (Noncalculator)

Given the parametric equations $x = 2\sqrt{t}$ and $y = 3t^2 - 2t$, find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$

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Ex. 2 (Noncalculator)

Given the parametric equations $x = 4\cos t$ and $y = 3\sin t$ write an equation of the tangent line to the curve at the point where $t = \frac{3\pi}{4}$.

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Ex 3 (Noncalculator)

Find all points of horizontal and vertical tangency given the parametric equations $x = t^2 + t,\ y = t^2 - 3t + 5.$

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Ex 4

Sketch the graph, indicate the direction, find the lowest point, and find any points of inflection.

$$x = t^2 - 3 \qquad 0 \le t \le \pi$$
$$y = -4\sin t$$

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Arc Length

$$\int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

from t = a to t = b, or distance traveled by the particle from a to b

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Ex. 5 (Noncalculator)

Set up an integral expression for the arc length of the curve given by the parametric equations $x = t^2 + 1,\ y = 4t^3 - 1,\ 0 \le t \le 1.$

Do not evaluate.

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Ex 6: Find the arc length:

$$x = 2\cos^3 t$$
$$y = 2\sin^3 t \qquad 0 \le t \le \pi$$

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