8.3

3. $\lim\limits_{x\to\infty} \dfrac{e^x}{e^{\cos x}} = \infty$

e^{-1} e^1

$\dfrac{1}{2.7182}$ $2.7182\cdot$

33.

3^x

$\sqrt{9^x+2^x} = \sqrt{3^{2x}} = 3^x$

$\sqrt{9^x-4^x} = \sqrt{3^{2x}} = 3^x$

$\lim\limits_{x\to\infty} \dfrac{\sqrt{3^{2x}+2^x}}{\sqrt{(3^x)^2}} = \dfrac{\sqrt{3^{2x}+2^x}}{\sqrt{3^{2x}}}$

$\sqrt{\dfrac{3^{2x}+2^x}{3^{2x}}}$

$\sqrt{\dfrac{3^{2x}}{3^{2x}} + \dfrac{2^x}{3^{2x}}}$

$\lim\limits_{x\to\infty} \sqrt{1 + \dfrac{2^x}{3^{2x}}} = 1$

39.

$$\lim_{x\to\infty} \frac{e^x}{x^n} \qquad \frac{e^x}{n\,x^{n-1}}$$

$$\frac{e^x}{x^3} \quad \frac{e^x}{3x^2} \quad \frac{e^x}{6x} \quad \frac{e^x}{6}$$

$$\frac{a^x}{x^n}$$

11.

$$\lim_{x\to\infty} \frac{\sqrt[3]{x^6+x^2}}{\sqrt[3]{(x^2)^3}}$$

$$\sqrt[3]{\frac{x^6+x^2}{x^6}}$$

$$\lim_{x\to\infty} \sqrt[3]{1+\frac{1}{x^4}} = 1$$

8.4a Improper Integrals

Evaluate using your calculator

$$\int_1^{100} \frac{1}{x}\,dx = \ln x \Big|_1^{100}$$

$$\ln 100 = \ln 10^2$$

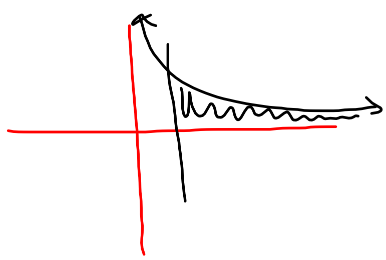
$$\int_1^{1000} \frac{1}{x}\,dx = 3 \ln 10$$

$$\int_1^{1{,}000{,}000} \frac{1}{x}\,dx = 6 \ln 10$$

Do you think $\boxed{\int_1^{\infty} \frac{1}{x}\,dx}$

converges or diverges?

If it converges, to what value does it converge?



Evaluate using your calculator

$$\frac{1}{e^x} = \left(\frac{1}{e}\right)^x$$

$$\int_1^{100} e^{-x}\,dx = e^{-1} - e^{-100}$$

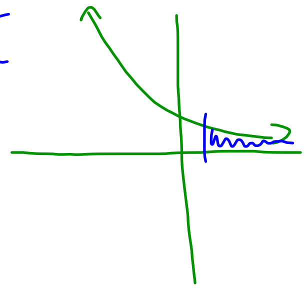
Do you think $\int_1^{\infty} e^{-x}\,dx$ converges or diverges?

$$\int_1^{1000} e^{-x}\,dx = e^{-1} - e^{-1000}$$

If it converges, to what value does it converge?

$$\int_1^{1000000} e^{-x}\,dx = \frac{1}{e} - \frac{1}{e^{1000000}}$$

$$\frac{1}{e}$$



1st type of improper integral: infinite limits

$$\int_{}^{\infty} \quad \text{or} \quad \int_{-\infty}^{} \quad \text{or} \quad \int_{-\infty}^{\infty}$$

has an ∞ or $-\infty$ in the bounds

$$\int_1^{\infty} \frac{1}{x}\,dx = \lim_{b\to\infty} \boxed{\int_1^{b} \frac{1}{x}\,dx}$$

$$\lim_{b\to\infty} \left(\ln x \Big|_1^b \right)$$

$$\lim_{b\to\infty} \left(\ln b - \ln 1 \right) = \infty$$

$$\int_1^\infty e^{-x}\,dx = \lim_{b\to\infty}\int_1^b e^{-x}\,dx = \lim_{b\to\infty}\left(-e^{-x}\Big|_1^b\right) =$$

$$\lim_{b\to\infty}\left(-e^{-b}+e^{-1}\right)$$

$$\lim_{b\to\infty}\left(\frac{-1}{e^b}+\frac{1}{e}\right)$$

$$0+\boxed{\frac{1}{e}}$$

$$\int_1^\infty xe^{-x}\,dx = \lim_{b\to\infty}\int_1^b xe^{-x}\,dx$$

x		e^{-x}
1	$+$	$-e^{-x}$
0	$-$	e^{-x}

$$\lim_{b\to\infty}\left(-xe^{-x}-e^{-x}\Big|_1^b\right)$$

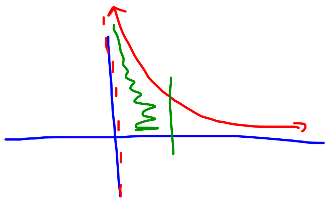
$$\lim_{b\to\infty}\left(-be^{-b}-e^{-b}-\left(-1e^{-1}-e^{-1}\right)\right)$$

$$\lim_{b\to\infty}\left(\cancel{\frac{-b}{e^b}}^{\,0}-\cancel{\frac{1}{e^b}}^{\,0}+\frac{1}{e}+\frac{1}{e}\right)$$

$$\frac{2}{e}$$

2nd type: one of the bounds is an asymptote

$$\int_0^1 \frac{1}{\sqrt{x}}\,dx$$



$$\lim_{b\to 0^+} \int_b^1 x^{-\frac{1}{2}}\,dx$$

$$\lim_{b\to 0^+} \left(2x^{\frac{1}{2}} \Big|_b^1 \right)$$

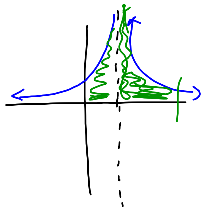
$$\lim_{b\to 0^+} \left(2 - 2b^{\frac{1}{2}}\right)$$

$$2$$

3rd type: there is an asymptote within the bounds of the limit

$$\int_0^3 \frac{1}{(x-1)^{\frac{2}{3}}}\,dx$$

asy @ $x=1$



$$\int_0^1 + \int_1^3$$

$$\lim_{b\to 1^-} \int_0^b (x-1)^{-\frac{2}{3}}\,dx + \lim_{b\to 1^+} \int_b^3 (x-1)^{-\frac{2}{3}}\,dx$$

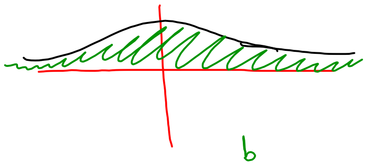
$$\lim_{b\to 1^-} \left(3(x-1)^{\frac{1}{3}} \Big|_0^b \right) + \lim_{b\to 1^+} \left(3(x-1)^{\frac{1}{3}} \Big|_b^3 \right)$$

$$\lim_{b\to 1^-} \left(\cancel{3(b-1)^{\frac{1}{3}}}^{\,0} - 3(-1)^{\frac{1}{3}} \right) + \lim_{b\to 1^+} \left(3(2)^{\frac{1}{3}} - \cancel{3(b-1)^{\frac{1}{3}}}^{\,0} \right)$$

$$3 + 3(2)^{\frac{1}{3}}$$

$$3 + 3\sqrt[3]{2}$$

$$\int_{-\infty}^{\infty} \frac{1}{1+x^2}\,dx$$

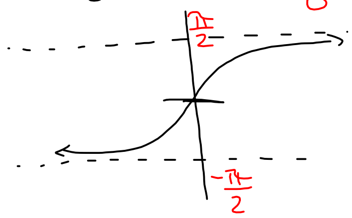


$$\lim_{b\to-\infty}\int_b^0 \frac{1}{1+x^2}\,dx + \lim_{b\to\infty}\int_0^b \frac{1}{1+x^2}\,dx$$

$$\lim_{b\to-\infty}\left(\tan^{-1}x\Big|_b^0\right) + \lim_{b\to\infty}\left(\tan^{-1}x\Big|_0^b\right)$$

$$\lim_{b\to-\infty}\left(\tan^{-1}(0) - \tan^{-1}b\right) + \lim_{b\to\infty}\left(\tan^{-1}b - \tan^{-1}0\right)$$

$$0 - \left(\frac{-\pi}{2}\right) \quad + \quad \frac{\pi}{2}$$



$$\frac{\pi}{2} + \frac{\pi}{2} = \pi$$