8.1

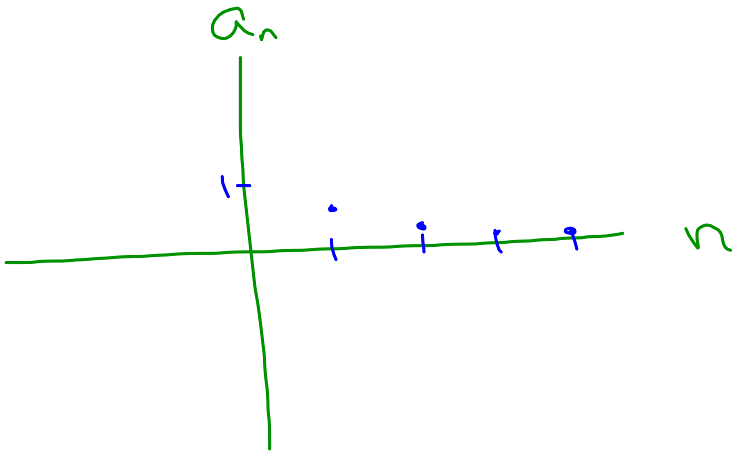
10. $V_1 = -3$

$V_2 = 2$

$V_n = V_{n-1} + V_{n-2}$

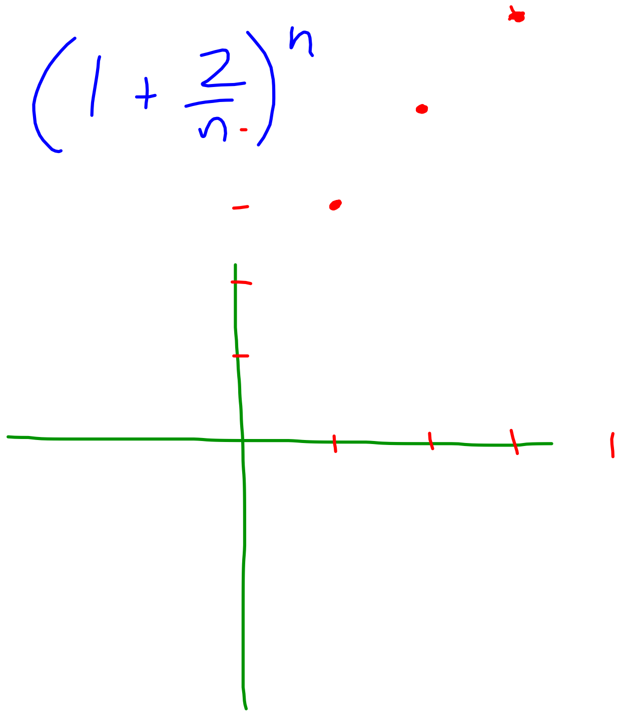
$V_3 = V_2 + V_1$

23. $a_n = \dfrac{n}{n^2+1}$



26.

$$a_n = \left(1 + \frac{2}{n}\right)^n$$



41.

$$a_n = \frac{\sin n}{n}$$

$$\overset{\to 0}{\frac{-1}{n}} \le \frac{\sin n}{n} \le \overset{\to 0}{\frac{1}{n}}$$

17c.

$-3 \quad 9 \quad -27 \quad 81$

$r = -3$

$$\begin{cases} a_1 = -3 \\ a_{n+1} = -3(a_n) \end{cases}$$

43.

$$\frac{1}{n!}$$

$$0 \le \frac{1}{n!} \le \frac{1}{n} \to 0$$

8.2

38.

$$\lim_{x\to 0^+} (\ln x - \ln(\sin x))$$

$$\lim_{x\to 0^+} \ln\left(\frac{x}{\sin x}\right) = 0$$

$$\lim_{x\to 0^+} \frac{1}{\cos x} = 1$$

24.

$$\lim_{x\to 0^+} \ln(\sin x)^{x} = \ln y \quad (0^0)$$

$$\lim_{x\to 0^+} x \ln \sin x = \ln y$$

$$\lim_{x\to 0^+} \frac{\ln \sin x}{\frac{1}{x}} = \ln y$$

$$\lim_{x\to 0^+} \frac{\frac{1}{\sin x} \cdot \cos x}{-\frac{1}{x^2}} = \ln y$$

$$\lim_{x\to 0^+} -\frac{x^2 \cos x}{\sin x} = \ln y$$

$$\lim_{x\to 0^+} \frac{-x^2(-\sin x) + \cos x(-2x)}{\cos x} = \ln y$$

$$0 = \ln y$$

$$y = 1$$

29.

$$\lim_{\theta \to 0} \frac{\sin 3\theta}{\sin 4\theta}$$

$$\lim_{\theta \to 0} \frac{3 \cdot \cos 3\theta}{4 \cdot \cos 4\theta} = \frac{3}{4}$$

8.3 Relative Rates of Growth

Definitions: Faster, Slower, Same-rate Growth

if $\displaystyle\lim_{x\to\infty} \frac{f(x)}{g(x)} = \infty$ f(x) grows faster

$\displaystyle\lim_{x\to\infty} \frac{f(x)}{g(x)} = 0$ g(x) grows faster

x^2 $3x^2$

$\displaystyle\lim_{x\to\infty} \frac{f(x)}{g(x)} = L$ $0 < L < \infty$

$3x^2 \quad (1, 3) \quad (2, 12)$

$\frac{9}{1}$

$x^2 \quad (1, 1) \quad (2, 4)$

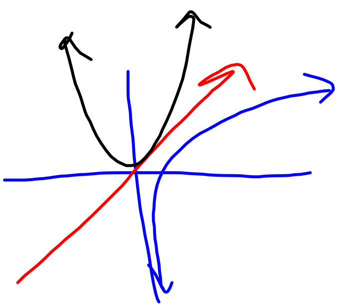
$\frac{3}{1}$

Which function grows faster?

$e^x \qquad x^2$

$$\lim_{x\to\infty} \frac{x^2}{e^x} = \lim_{x\to\infty} \frac{2x}{e^x} = \lim_{x\to\infty} \frac{2}{e^x} = 0$$

$x \qquad \ln x \qquad x^2$



Show the functions grow at the same rate:

$x,\ x+\sin x$

$$\lim_{x\to\infty}\frac{x}{x+\sin x} = \lim_{x\to\infty}\frac{1}{1+\cos x}$$

$$\lim_{x\to\infty}\frac{x+\sin x}{x} = \lim_{x\to\infty}\frac{1+\cos x}{1} =$$

$$0 \leq 1+\cos x \leq 2$$

$\log_a x,\ \log_b x$

$$\lim_{x\to\infty}\frac{\log_a x}{\log_b x} = \lim_{x\to\infty}\frac{\frac{1}{\cancel{x}\ln a}}{\cancel{\frac{1}{x\ln b}}} \quad \cdot\cancel{x}\ln b$$

$$\lim_{x\to\infty}\frac{\ln b}{\ln a}$$

Transitivity of Growing Rates:

$a=b \quad b=c \quad$ then $\quad a=c$

Show the functions grow at the same rate by comparing both with x

$\sqrt{x^2+5} \qquad \left(2\sqrt{x}-1\right)^2$

$$\lim_{x\to\infty}\frac{\sqrt{x^2+5}}{x} = 1$$

$$\frac{2x^2+4x+5}{3x^2+2x-5}$$

$$\lim_{x\to\infty}\frac{\left(2\sqrt{x}-1\right)^2}{x}$$

$$\frac{2\left(2\sqrt{x}-1\right)^1\left(\cancel{2}\left(\tfrac{1}{\cancel{2}}x^{-\frac{1}{2}}\right)\right)}{1}$$

$$\lim_{x\to\infty}\frac{4\sqrt{x}-2}{\sqrt{x}}$$

$$\lim_{x\to\infty}\frac{2\cancel{x^{-\frac{1}{2}}}}{\frac{1}{2}\cancel{x^{-\frac{1}{2}}}} = 4$$