6.1 b

55.

$$\frac{dy}{dx} = e^{\frac{(x-y)}{2}}$$

$$-\frac{1}{e^{\frac{(x-y)}{2}}} = -e^{-\left(\frac{x-y}{2}\right)} = -e^{\frac{-x+y}{2}} = -e^{\frac{(y-x)}{2}}$$

65.

$$\frac{dy}{dx} = x - \frac{1}{x^2} \quad (x^{-2})$$

a. $(1,2)$

$$y = \frac{x^2}{2} + \frac{x^{-1}}{+1} + C$$

$$2 = \frac{1^2}{2} + 1^{-1} + C$$

$$1 = \frac{1}{2} + C \qquad y = \frac{x^2}{2} + x^{-1} + \frac{1}{2}$$

$$C = \frac{1}{2}$$

b. $(-1,1)$

$$1 = \frac{(-1)^2}{2} + (-1)^{-1} + C$$

$$2 = \frac{1}{2} + C \qquad y = \frac{x^2}{2} + x^{-1} + \frac{3}{2}$$

$$C = \frac{3}{2}$$

c.

$$f(x) = \begin{cases} \frac{x^2}{2} + \frac{1}{x} + C_1 & x < 0 \\ \frac{x^2}{2} + \frac{1}{x} + C_2 & x > 0 \end{cases}$$

$$f'(x) = x - \frac{1}{x^2}$$

d. $C_1 = \frac{3}{2}$

$C_2 = \frac{1}{2}$

67.

a. $\frac{d^2y}{dx^2} = 12x + 4$

$\frac{dy}{dx} = 6x^2 + 4x + C_1$

$y = 2x^3 + 2x^2 + C_1 x + C_2$

6.2a Integration by Substitution U- substitution

A change of variables can turn an unfamiliar integral into one that we can evaluate. (The differential matters.)

$$\int f(x)dx = \int g(u)du$$

$$\int \sin(x)e^{\cos(x)}dx$$

$u = \cos x$

$\frac{du}{-\sin x} = \frac{\cancel{-\sin x}\,dx}{\cancel{-\sin x}}$

$\int \cancel{\sin x}\, e^u \frac{du}{-\cancel{\sin x}} = \int -e^u du$

$-\int e^u du$

$= -e^u + C$

$= -e^{\cos x} + C$

$-e^{\cos x}(-\sin x)$

$$\int 5\cos 5x\,dx$$

$u = 5x$

$\frac{du}{5} = \cancel{5}\,dx$

$$\int \cancel{5}\cos u \frac{du}{\cancel{5}}$$

$$\int \cos u\,du$$

$$= \sin u + C$$

$$= \sin(5x) + C$$

$$\int 2x(x^2+1)^4\,dx$$

$u = x^2+1$

$du = 2x\,dx$

$$\int u^4\,du$$

$$= \frac{u^5}{5} + C$$

$$= \frac{(x^2+1)^5}{5} + C$$

$$\frac{1}{6}\int 6x^2\sqrt{5+2x^3}\,dx$$

$u = 5+2x^3$

$\frac{du}{6x^2} = 6x^2\,dx$

$$\int \cancel{x^2}\, u^{\frac{1}{2}} \frac{du}{6\cancel{x^2}}$$

$$\frac{1}{6}\int u^{\frac{1}{2}}\,du$$

$$\frac{1}{6}\int u^{\frac{1}{2}}\,du$$

$$= \frac{1}{6}\left(\frac{2}{3}u^{\frac{3}{2}}\right) + C$$

$$= \frac{1}{9}(5+2x^3)^{\frac{3}{2}} + C$$

$\int \cot(7x)dx$

$\int \frac{\cos(7x)}{\sin(7x)}dx$

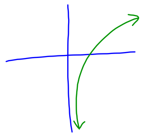
$u = \sin 7x$

$\frac{du}{7\cos 7x} = \cancel{7\cos 7x}\,dx$

$\int \frac{\cancel{\cos(7x)}}{u} \cdot \frac{du}{7\cancel{\cos 7x}}$

$\frac{1}{7}\int \frac{1}{u}\,du$

$\frac{1}{7}\ln|u| + C$



$= \frac{1}{7}\ln|\sin 7x| + C$

$\int \tan(5x)dx$

$\int \frac{\sin(5x)}{\cos(5x)}dx$

$u = \cos 5x$

$\frac{du}{-5\sin 5x} = -\cancel{5\sin 5x}\,dx$

$\int \frac{\cancel{\sin(5x)}}{u} \cdot \frac{du}{(-5\cancel{\sin 5x})}$

$-\frac{1}{5}\int \frac{1}{u}\,du$

$= -\frac{1}{5}\ln|\cos 5x| + C$

$$\int \frac{dx}{\cos^2 2x}$$

$\int \sec^2(2x)\,dx$

$u = 2x$

$\frac{du}{2} = \cancel{2}\,dx$

$= \frac{\tan(2x)}{2} + C$

$$\int \cot^2(3x)\,dx$$

$$\sin^2\theta = \frac{1-\cos 2\theta}{2}$$

$$\frac{\cos^2(3x)}{\sin^2(3x)}$$

$$\cos^2\theta = \frac{1+\cos 2\theta}{2}$$

$\overline{\sin^2 x}$ $\overline{\sin^2 x}$ $\overline{\sin^2 x}$

$$\frac{\sin^2 x}{\cos^2 x} + \frac{\cos^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$$

$$* \; 1 + \cot^2 x = \csc^2 x$$

$$\tan^2 x + 1 = \sec^2 x$$

$$\int \left(\csc^2(3x) - 1\right) dx$$

$$= -\frac{\cot(3x)}{3} - x + C$$

$$\int \cos^3 x\,dx$$

$$\cos x\left(\cos^2 x\right)$$

$$\int \cos x\left(1-\sin^2 x\right) dx$$

$u = \sin x$

$du = \cos x\,dx$

$$\int \left(1-u^2\right) du$$

Definite Integrals:

$$\int_0^{\frac{\pi}{3}} \tan x \sec^2(x)\,dx$$

$$\int_0^1 \frac{x}{x^2-4}\,dx$$