10.1 & 10.2

$(3t)^{\frac{1}{2}}$

9. $x = -\sqrt{t+1}$ $y = \sqrt{3t}$

$$\frac{dy}{dx} = \frac{\frac{3}{2}(3t)^{-\frac{1}{2}}}{-\frac{1}{2}(t+1)^{-\frac{1}{2}}} = \frac{-3(t+1)^{\frac{1}{2}}}{(3t)^{\frac{1}{2}}}$$

$$= -3\sqrt{\frac{t+1}{3t}} = -3\sqrt{\frac{1}{3}+\frac{1}{3t}}$$

$$\frac{t}{3t} + \frac{1}{3t}$$

$$\frac{d^2y}{dx^2} = \frac{+\frac{3}{2}\left(\frac{1}{3}+\frac{1}{3t}\right)^{-\frac{1}{2}} \cdot \frac{-3}{(3t)^2}}{+\frac{1}{2}(t+1)^{-\frac{1}{2}}}$$

$$\frac{-9}{9t^2}\frac{(t+1)^{\frac{1}{2}}}{\left(\frac{1}{3}+\frac{1}{3t}\right)^{\frac{1}{2}}} = \frac{-(t+1)^{\frac{1}{2}}}{t^2\left(\frac{1}{3}+\frac{1}{3t}\right)^{\frac{1}{2}}}$$

15.
16.

23.

15. $x = \ln(2t)$

$y = \ln(3t)^4 = 4\ln(3t)$

$$\frac{dy}{dt} = \frac{1}{(3t)^4} \cdot 4(3t)^3 \cdot 3$$

$$4\left(\frac{1}{3t}\right)^3$$

$$\frac{12}{3t} = \frac{4}{t}$$

$$\frac{dx}{dt} = \frac{1}{2t} \cdot 2 = \frac{1}{t}$$

$$\frac{dy}{dx} = \frac{\frac{4}{t}}{\frac{1}{t}} = 4$$

$$\frac{d^2y}{dx^2} = \frac{0}{\frac{1}{t}} = 0$$

16.

$x = \ln(5t) \qquad y = e^{5t}$

$\frac{dy}{dx} = \frac{5e^{5t}}{\frac{1}{t}} = (5t)(e^{5t})$

$\frac{d^2y}{dx^2} = \frac{5t(5e^{5t}) + 5e^{5t}}{\frac{1}{t}}$

$= 25t^2e^{5t} + 5te^{5t}$

23.

$x = 2 + \cos t \qquad y = -1 + \sin t$

$\frac{dy}{dx} = \frac{\cos t}{-\sin t}$

vertical:

$-\sin t = 0$

$t = 0, \pi$

$(3,-1)\ (1,-1)$

horizontal:

$\cos t = 0$

$t = \frac{\pi}{2}, \frac{3\pi}{2}$

$(2,0)\ (2,-2)$

10.2

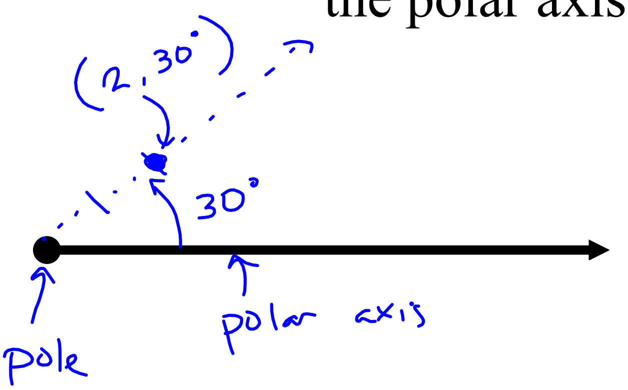
10.3 Polar Functions

Polar Points: (r, θ)

r: directed distance from the pole

θ: directed angle measured from the polar axis

$(2, 30°)$

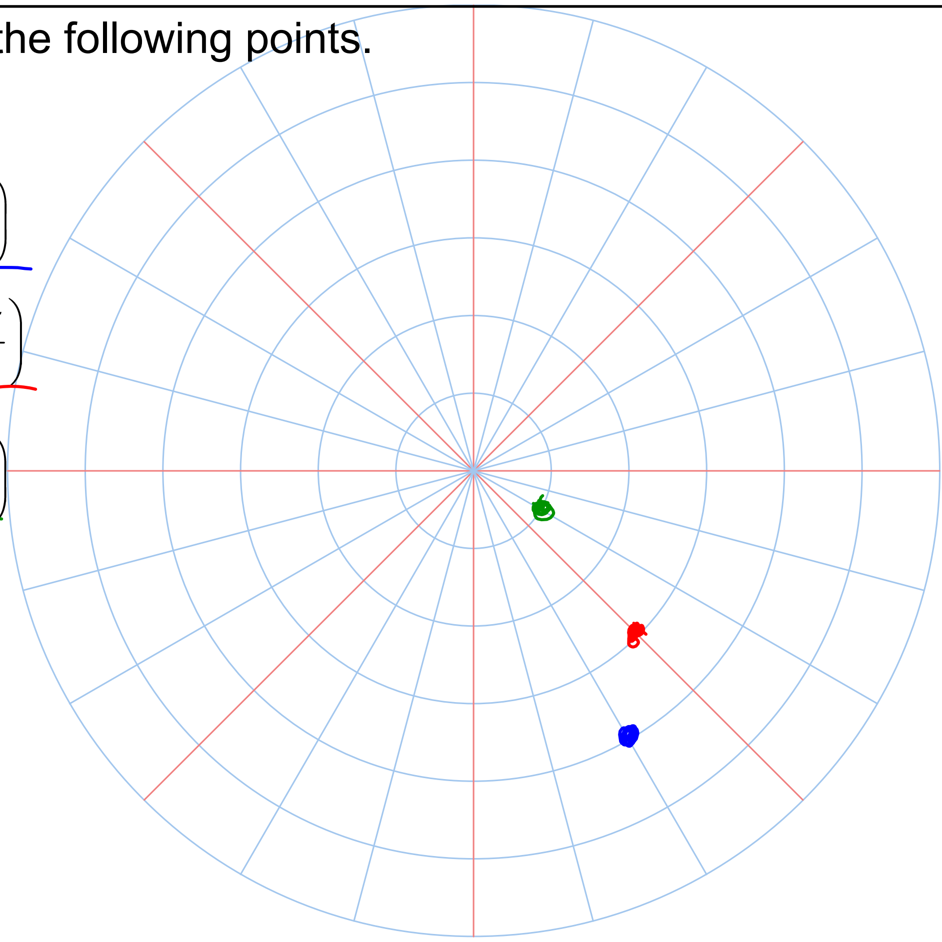


Graph the following points.

$\left(4, \frac{5\pi}{3}\right)$

$\left(-3, \frac{3\pi}{4}\right)$

$\left(1, -\frac{\pi}{6}\right)$



Conversion: Rectangular to Polar

$(x, y) \rightarrow (r, \theta)$

Conversion: Polar to Rectangular

$(r, \theta) \rightarrow (x, y)$

$x^2 + y^2 = r^2$

$\cos\theta = \frac{x}{r}$

$\sin\theta = \frac{y}{r}$

$\tan\theta = \frac{y}{x}$

$x = r\cos\theta$

$y = r\sin\theta$

Give the polar coordinates for:

(2, 2)

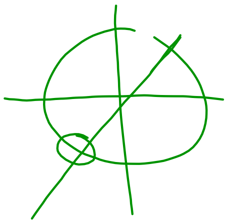
$2^2 + 2^2 = 8$

$r = \pm\sqrt{8}$

$\tan\left(\frac{2}{2}\right) = \tan(1) = 45^\circ$

$\left(2\sqrt{2}, \frac{\pi}{4}\right)$

$\left(-2\sqrt{2}, \frac{5\pi}{4}\right)$



(3, -3)

$\tan^{-1}\left(\frac{-3}{3}\right) = -\frac{\pi}{4}$

$3^2 + 3^2 = \sqrt{18}$

$3\sqrt{2}$

$\left(3\sqrt{2}, \frac{7\pi}{4}\right)$

$\left(3\sqrt{2}, -\frac{\pi}{4}\right)$

$\left(-3\sqrt{2}, \frac{3\pi}{4}\right)$

Give the rectangular coordinates for:

$\left(2, \frac{5\pi}{6}\right)$

$2\cos\frac{5\pi}{6} = x$

$2\sin\frac{5\pi}{6} = y$

$\left(-\sqrt{3}, 1\right)$

Conversions with Polar Equations

to convert equations use:

$$x = r\cos\theta \qquad x^2 + y^2 = r^2$$
$$y = r\sin\theta$$

and other identities as needed

y = 4

$r\sin\theta = 4$

$r = \frac{4}{\sin\theta}$

$2x^2 + 2y^2 = 25$

$(x^2 + y^2) = \frac{25}{2}$

$\sqrt{r^2} = \sqrt{\frac{25}{2}}$ $r = \pm\frac{5}{\sqrt{2}}$

$r\sin\theta = 3$

$y = 3$

$r \cdot r = 2r\cos\theta$

$x^2 + y^2 = 2x$

$\theta = \frac{\pi}{6}$

$\tan\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{3}$

$m = \frac{\sqrt{3}}{3}$

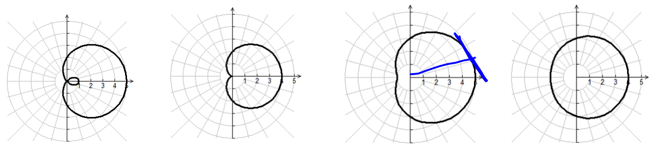
$y = \frac{\sqrt{3}}{3}x$

Limaçons

$r = a \pm b\cos\theta$

$r = a \pm b\sin\theta$

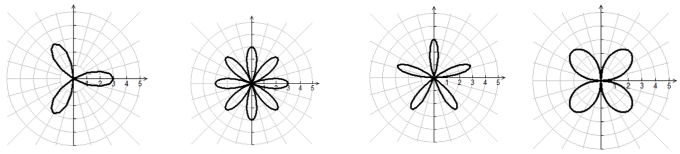
$a > 0, b > 0$



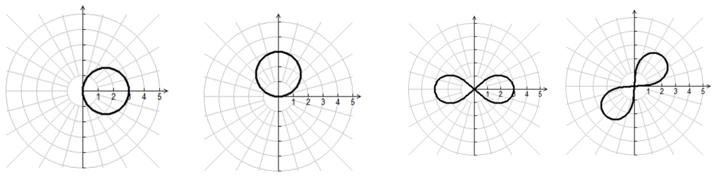
Rose Curves

n petals if n is odd
2n petals if n is even

$n \geq 2$



Circles and Lemniscates



Calculus with Polar Equations

$x = r\cos\theta$

$y = r\sin\theta$

$$\frac{dy}{d\theta} = r\cos\theta + \sin\theta \cdot \frac{dr}{d\theta}$$

slope:

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$$

Or

$$\frac{dy}{dx} = \frac{r'\sin\theta + r\cos\theta}{r'\cos\theta - r\sin\theta}$$

Find $\frac{dy}{dx}$ and the slope of the graph at the given value:

$r = 3 + 2\sin\theta$ $\quad \theta = \frac{\pi}{6}$

$x = (3 + 2\sin\theta)\cos\theta$

$y = (3 + 2\sin\theta)\sin\theta$

$$\frac{dy}{dx} = \frac{(3+2\sin\theta)\cos\theta + \sin\theta(2\cos\theta)}{(3+2\sin\theta)(-\sin\theta) + \cos\theta(2\cos\theta)}$$

$$\left.\frac{dy}{dx}\right|_{\theta=\frac{\pi}{6}} = \frac{(4)\left(\frac{\sqrt{3}}{2}\right) + \frac{1}{2}(\sqrt{3})}{(4)\left(-\frac{1}{2}\right) + \frac{\sqrt{3}}{2}(\sqrt{3})}$$

$$= \frac{\frac{5\sqrt{3}}{2}}{-\frac{1}{2}} = -5\sqrt{3}$$