

$$\frac{dx}{dt} = -\frac{3}{4} \frac{dy}{dt}$$

$$x^2 + y^2 = 40^2 \quad x^2 + \left(\frac{3}{4}x\right)^2 = 40^2$$

$$2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 0$$

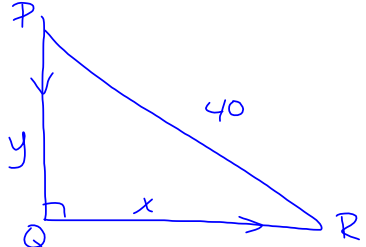
$$x\left(-\frac{3}{4} \frac{dy}{dt}\right) + y \frac{dy}{dt} = 0$$

$$\frac{dy}{dt} \left(-\frac{3x}{4} + y\right) = 0$$

$$-\frac{3}{4}x + y = 0$$

$$y = \frac{3}{4}x$$

34



$$x^2 + y^2 = 40^2 \quad \frac{dx}{dt} = -\frac{3}{4} \frac{dy}{dt}$$

$$2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 0 \quad x^2 + \left(\frac{3}{4}x\right)^2 = 40^2$$

$$x\left(-\frac{3}{4} \frac{dy}{dt}\right) + y \frac{dy}{dt} = 0$$

$$\frac{dy}{dt} \left(-\frac{3}{4}x + y\right) = 0$$

$$-\frac{3}{4}x + y = 0$$

$$y = \frac{3}{4}x$$

36.

$$h + 2\pi r = 30 \text{ cm}$$

$$V = h\pi r^2$$

$$h = 30 - 2\pi r$$

$$V = \pi r^2 (30 - 2\pi r)$$

$$V = \pi (30r^2 - 2\pi r^3)$$

$$\frac{dV}{dr} = \pi (60r - 6\pi r^2) = 0$$

$$60r\pi (10 - \pi r) = 0$$

$$60r\pi = 0$$

$$r = 0$$

$$10 - \pi r = 0$$

$$r = \frac{10}{\pi}$$

37.

$$\int_0^1 x \, dx + \int_1^e \frac{1}{x} \, dx$$

$$\frac{x^2}{2} \Big|_0^1 + \ln x \Big|_1^e$$

$$\frac{1}{2} + \ln e - \ln 1$$

38.

$$\frac{dP}{dt} = kP$$

$$\frac{dy}{dt} = ky$$

$$\frac{dy}{y} = k dt$$

$$\ln y = kt + C$$

$$y = e^{kt+C}$$

$$y = C e^{kt}$$

$$y = y_0 e^{kt}$$

$$y = 1000 e^{kt}$$

$$1200 = 1000 e^{k(7)}$$

$$\ln\left(\frac{6}{5}\right) = 7k$$

$$\frac{\ln\left(\frac{6}{5}\right)}{7} = k$$

39.

$$\int \frac{dy}{dx} = \int \frac{1}{x} dx$$

$$y = \ln x$$

[1, 4]
(1, 0)
(4, ln 4)

avg. rate of change = $\frac{\ln 4 - 0}{4 - 1} = \frac{\ln 4}{3}$

$$\frac{\ln 2^2}{3}$$

41.

$$f(x) = \int_{-2}^{x^2-3x} e^{t^2} dt$$

$$f'(x) = e^{(x^2-3x)^2} (2x-3) = 0$$

$$e^{(x^2-3x)^2} = 0 \quad 2x-3 = 0$$

$$x = \frac{3}{2}$$

42.

$$\lim_{x \rightarrow 0} \ln(1+2x)^{\csc x} = \ln y$$

$$\lim_{x \rightarrow 0} (\csc x)(\ln(1+2x)) = \ln y$$

$$\lim_{x \rightarrow 0} \frac{\ln(1+2x)}{\sin x} = \ln y$$

$$\lim_{x \rightarrow 0} \frac{\frac{1}{1+2x} \cdot 2}{\cos x} = \ln y$$

$$\frac{2}{1} = \ln y$$

$$e^2 = y$$

44.

$$\frac{1}{b-a} \int_a^b f(x) dx = f(c)(b-a)$$

45.

$$\sum_{n=1}^{\infty} (\sin^2 1)^n$$

 $f(1)$

$$S = \frac{\sin^2(1)}{1 - \sin^2(1)}$$

36.

$$V = \pi r^2 h$$

$$h + 2\pi r = 30$$

$$V = \pi r^2 (30 - 2\pi r)$$

$$h = 30 - 2\pi r$$

$$V = 30\pi r^2 - 2\pi^2 r^3$$

$$\frac{dV}{dr} = 60\pi r - 6\pi^2 r^2 = 0$$

$$6\pi r(10 - \pi r) = 0$$

$$6\pi r = 0$$

$$10 - \pi r = 0$$

$$r = 0$$

$$r = \frac{10}{\pi}$$

39.

$$\frac{dy}{dx} = \frac{1}{x}$$

$$dy = \frac{1}{x} dx$$

$$y = \ln x \quad [1, 4]$$

$$\frac{\ln 4 - \ln 1}{4 - 1} = \frac{\ln 4}{3} = \frac{\ln 2^2}{3}$$

$$\frac{dy}{dt} = ky$$

$$y = y_0 e^{kt}$$

$$\frac{dy}{y} = k dt$$

$$y = 1000 e^{kt}$$

$$\ln y = kt + C$$

$$1200 = 1000 e^{k \cdot 7}$$

$$e^{kt+C} = y$$

$$\ln \frac{6}{5} = \ln e^{7k}$$

$$C e^{kt} = y$$

$$y = y_0 e^{kt}$$

$$\frac{\ln \frac{6}{5} = 7k}{7}$$

41.

$$f(x) = \int_{-2}^{x^2-3x} e^{t^2} dt$$

$$e^{(x^2-3x)^2} \cdot (2x-3) = 0$$

$$e^{(x^2-3x)^2} = 0 \quad 2x-3=0$$

$$\frac{\int_a^b f(x) dx}{\cancel{b-a}} = f(c) (b-a)$$

$$\frac{1}{\cancel{b-a}} \int_a^b f(x) dx = f(c) (b-a)$$

$$(\sin^2 x)^k$$

$$\sum_{n=1}^{\infty} (\sin^2 x)^n$$

$$S = \frac{(\sin^2(1))^1}{1 - \sin^2(1)}$$

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|----|---|-----|---|-----|---|
| 7. | C | 14. | C | | |
| 1. | C | 8. | E | 15. | D |
| 2. | E | 9. | A | 16. | B |
| 3. | A | 10. | B | 17. | B |
| 4. | C | 11. | C | 18. | C |
| 5. | C | 12. | A | 19. | D |
| 6. | A | 13. | B | 20. | E |

21. A

22. C

23. E

24. D

25. A

$$1. \int_0^1 \sqrt{x}(x+1) dx$$
$$\int_0^1 x^{\frac{1}{2}}(x+1) dx$$
$$\int_0^1 x^{\frac{3}{2}} + x^{\frac{1}{2}} dx$$

$$1. \int_0^1 \sqrt{x}(x+1) dx$$
$$\int_0^1 x^{\frac{3}{2}} + x^{\frac{1}{2}} dx$$
$$\frac{2}{5} x^{\frac{5}{2}} + \frac{2}{3} x^{\frac{3}{2}} \Big|_0^1$$

$$2. \quad x = e^{2t} \quad y = \sin(2t)$$

$$\frac{dy}{dx} = \frac{\cancel{2} \cos(2t)}{\cancel{2} e^{2t}}$$

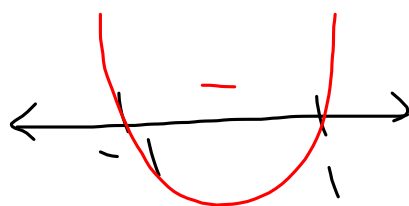
3.

$$f'(x) = 15x^4 - 12x^2 - 3 = 0$$

$$3(5x^4 - 4x^2 - 1) = 0$$

$$3(5x^2 + 1)(x^2 - 1) = 0$$

$$x = \pm 1$$



6.

$$y = \sqrt{16-x}$$

$$\frac{dy}{dx} = \frac{1}{2} (16-x)^{-\frac{1}{2}} (-1) \quad \Big|_{(0,4)}$$

$$= \frac{1}{2} \left(\frac{1}{4} \right) = -\frac{1}{8}$$

10.

$$y = xy + x^2 + 1$$

$$y = -y + 1 + 1$$

$$\frac{x = -1}{y = 1}$$

$$\frac{dy}{dx} = x \frac{dy}{dx} + y + 2x^{(-1)}$$

$$2 \frac{dy}{dx} = y - 2$$

11.

$$\int_{-1}^{\infty} \frac{x}{(1+x^2)^2} dx$$

$$u = 1+x^2$$

$$\frac{du}{2x} = 2x dx$$

$$\int \frac{x}{u^2} \cdot \frac{du}{2x}$$

$$\frac{1}{2} \int u^{-2} du$$

$$\frac{1}{2} \frac{u^{-1}}{-1}$$

$$\lim_{b \rightarrow \infty} \left(\frac{-1}{2(1+x^2)} \Big|_{-1}^b \right)$$

$$\lim_{b \rightarrow \infty} \left(\frac{-1}{2(1+b^2)} - \frac{-1}{2(1+1)} \right)$$

$$\frac{1}{4}$$

13.

$$a(t) = 2t - 7$$

$$v(0) = 6$$

$$v(t) = 6 + \int_0^t 2x - 7 dx$$

$$v(t) = t^2 - 7t + 6 = 0$$

$$(t-6)(t-1) = 0$$

$$t = \cancel{6}, 1$$

14.

$$S = \frac{\frac{3}{2}}{1 - \frac{3}{8}} = \frac{\frac{3}{2} \cdot \frac{8}{5}}{\frac{5}{8}} = \frac{24}{10}$$

15.

$$\int_0^{2\pi} \sqrt{(3\cos^2 t (-\sin t))^2 + (3\sin^2 t \cdot \cos t)^2} dt$$

$$\int_0^{2\pi} \sqrt{9\cos^4 t \sin^2 t + 9\sin^4 t \cos^2 t}$$

17.

$$f(a) + \frac{f'(a)(x-a)}{1!} + \frac{f''(a)(x-a)^2}{2!}$$

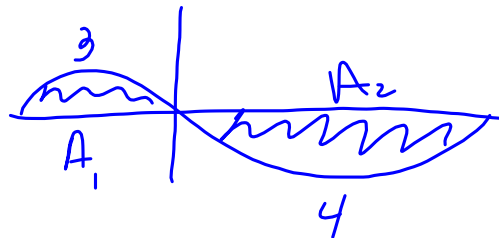
$$0 + \frac{-1(x-2)}{1!} + \frac{-1(x-2)^2}{2}$$

$$f(x) = \ln(3-x)$$

$$f'(x) = \frac{1}{3-x} \cdot -1 = -(3-x)^{-1}$$

$$f''(x) = (3-x)^{-2} \cdot -1$$

19.



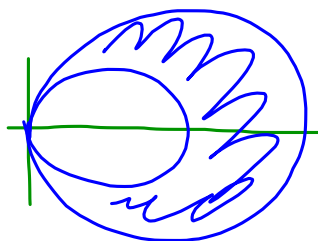
$$\int_{-4}^4 f(x) dx = 2 \int_{-1}^4 f(x) dx$$

$$A_1 - A_2 = 2(-A_2)$$

21.

$$r = 2 \cos \theta$$

$$r = \cos \theta$$



3.

$$3x^5 - 4x^3 - 3x$$

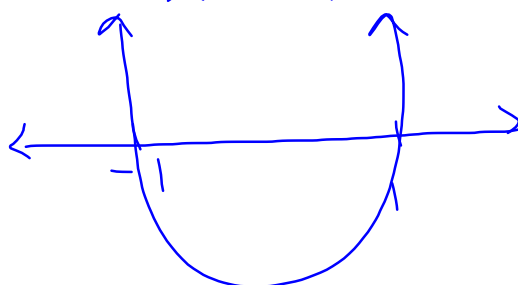
$$f'(x) = 15x^4 - 12x^2 - 3$$

$$3(5x^4 - 4x^2 - 1)$$

$$3(5x^2 + 1)(x^2 - 1) = 0$$

(x+1)(x-1)

$$x = \pm 1$$



$$6. \quad y = \sqrt{16-x} \quad (0, 4)$$

$$y' = \frac{1}{2} (16-x)^{-\frac{1}{2}} (-1)$$

$$y' = -\frac{1}{2} \left(\frac{1}{4} \right) = -\frac{1}{8}$$

10.

$$y = xy + x^2 + 1$$

$$y = 1$$

$$x = -1$$

$$y = -1y + (-1)^2 + 1$$

$$2y = 2$$

$$y = 1$$

$$\frac{dy}{dx} = x \frac{dy}{dx} + y + 2x$$

$$\frac{dy}{dx} = -\frac{dy}{dx} + 1 + 2(-1)$$

$$2 \frac{dy}{dx} = -1$$

$$13. \quad a = 2t - 7$$

$$v(0) = 6$$

$$v(t) = 6 + \int_0^t (2x - 7) dx$$

$$v(t) = 6 + \left(x^2 - 7x \right) \Big|_0^t$$

$$v(t) = t^2 - 7t + 6 = 0$$

$$(t - 6)(t - 1) = 0$$

$$t = 1, \cancel{6}$$

14.

$$\frac{3}{2} + \frac{9}{16} + \frac{27}{128}$$

$$r = \frac{3}{8}$$

$$S = \frac{\frac{3}{2}}{1 - \frac{3}{8}}$$

$$17. \quad f(a) + \frac{f'(a)(x-a)}{1!} + \frac{f''(a)(x-a)^2}{2!}$$

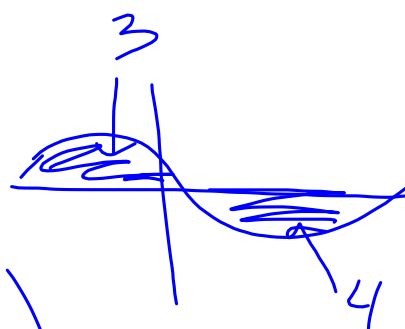
$$\ln(3-x)$$

$$0 + \frac{-(x-2)}{1!} + \frac{-1(x-2)^2}{2!}$$

$$f' = \frac{1}{3-x} \cdot -1 = -1(3-x)^{-1}$$

$$f'' = 1(3-x)^{-2}(-1) = \frac{-1}{(3-x)^2}$$

$$19. \quad \int_{-4}^4 f(x) dx - 2 \int_{-1}^4 f(x) dx$$



$$A_1 - A_2 - 2(-A_2)$$

$$(x+2)(x-3)=0$$