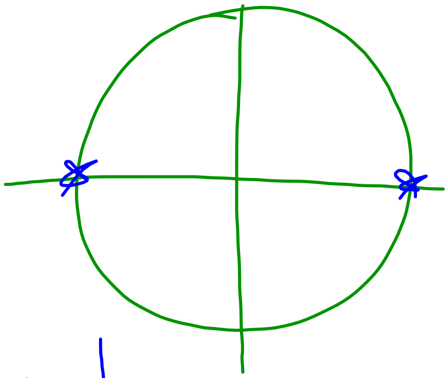
9.5

31. $\sum_{n=1}^{\infty} \left| \frac{\cos n\pi}{n} \right|$



$$\frac{-1}{1} + \frac{1}{2} + \frac{-1}{3} + \frac{1}{4} + \cdots$$

$$\frac{1}{n}$$

Direct Comparison

$$\sum_{n=1}^{\infty} \frac{1}{3^n+2}$$

$$\frac{1}{3^n+2} < \frac{1}{3^n}$$

geo. $r = \frac{1}{3}$

conv.

$\therefore \frac{1}{3^n+2}$ conv. by Dir. Comp. Test

$$\sum_{n=1}^{\infty} \frac{1}{n^3+2}$$

$$\sum_{n=1}^{\infty} \frac{5}{n^3+2} = 5 \sum_{n=1}^{\infty} \frac{1}{n^3+2}$$

Review

3. $\sum_{n=0}^{\infty} \left(\frac{2}{3}\right)^n (x-1)^n = \sum_{n=0}^{\infty} \left(\frac{2}{3}(x-1)\right)^n$

$$r = \frac{2}{3}(x-1)$$

$$-1 < \frac{2}{3}(x-1) < 1$$

$$-\frac{3}{2} < x-1 < \frac{3}{2}$$

$$-\frac{1}{2} < x < \frac{5}{2}$$

$$r = \frac{3}{2}$$

7.

$$\sum_{n=0}^{\infty} \frac{(n+1)(2x+1)^n}{(2n+1)2^n}$$

$$\frac{((n+1)+1)(2x+1)^{(n+1)}}{(2(n+1)+1)\,2^{(n+1)}} \cdot \frac{(2n+1)\cancel{2^n}}{(n+1)\cancel{(2x+1)^n}}$$

$$\lim_{n\to\infty} \frac{(n+2)(2n+1)(2x+1)}{(2n+3)(n+1)\,2} = \frac{2x+1}{2}$$

$$-1 < \frac{2x+1}{2} < 1$$

$$-2 < 2x+1 < 2$$

$$-\frac{3}{2} < \frac{2x}{2} < \frac{1}{2}$$

$$-\frac{3}{2} < x < \frac{1}{2}$$

$$\frac{(n+1)(2(\frac{-3}{2})+1)^n}{(2n+1)\,2^n}$$

$$\frac{(n+1)(-2)^n}{(2n+1)\,2^n}$$

$$\boxed{\frac{(n+1)(-1)^n}{(2n+1)}}\frac{\cancel{(2)^n}}{\cancel{2^n}}$$

n^{th} term $a_n \to \frac{1}{2} \neq 0$

Diverges

$$\frac{(n+1)(\cancel{2}(\frac{1}{\cancel{2}})+1)^n}{(2n+1)\cancel{(2^n)}}$$

Diverges n^{th} term

9.

$$\sum_{n=1}^{\infty} \frac{x^n}{\sqrt{n}}$$

$$\lim_{n\to\infty} \frac{x^{\cancel{n}+1}}{\sqrt{n+1}} \cdot \frac{\sqrt{n}}{\cancel{x^n}} = x$$

Endpts. $-1 \leq x < 1$

$$\frac{(-1)^n}{\sqrt{n}}$$

alt ✓

dec. in mag ✓ $\frac{1}{\sqrt{n}}$

$a_n \to 0$ ✓

Converges

$$\frac{(1)^n}{\sqrt{n}} = \frac{1}{n^{\frac{1}{2}}}$$

$p = \frac{1}{2}$

Diverges

13.

$$\sum_{n=1}^{\infty} \frac{n!}{2^n} x^{2n}$$

$$\frac{\cancel{(n+1)!}}{2^{\cancel{n}+1}} x^{\cancel{2n}+2} \cdot \frac{2^{\cancel{n}}}{\cancel{n!}\, \cancel{x^{2n}}}$$

$(n+1)\cancel{n!}$ $\quad 2(n+1)$

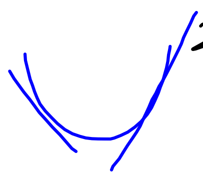
$$\lim_{n \to \infty} \frac{(n+1)x^2}{2} = \infty$$

$x = 0$

55.

inc $\quad$ concave up

$$P_3(x) = 1 + 4(x-3) + 6\frac{(x-3)^2}{2} + \frac{12(x-3)^3}{3!}$$



$$P_2(x) \approx f'(x) = 4 + \frac{12(x-3)}{2} + \frac{36(x-3)^2}{6}$$

Error:

$\boxed{20 \text{ terms}}$

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(2n+5)}{3^n}$$

$$|E| < \frac{(2(21)+5)}{3^{21}}$$

e^{2x}

use a 3rd order poly. to approx.

What's the Lagrange error Bound on the int.

$|x| < .5$

$$|E| \le \frac{M(x-0)^4}{4!}$$

$f = e^{2x}$

$f' = 2e^{2x}$

$f'' = 4e^{2x}$

$f''' = 8e^{2x}$

$f^4 = 16e^{2x(.5)}$

$$|E| \le \frac{16e(.5)^4}{4!}$$

19.

$$\pi - \frac{\pi^3}{3!} + \frac{\pi^5}{5!} \cdots$$

$\sin x$

$x = \pi$

$$S = \sin \pi = 0$$

55.

$$1 + \underline{4}(x-3) + \frac{\underline{6}(x-3)^2}{2!} + \frac{12(x-3)^3}{3!}$$

inc.

concave up

$$P_2(x) = f' \approx 4 + \frac{12(x-3)}{2!} + \frac{36(x-3)^2}{3!}$$

56.

$f(x) \approx P_4(x) = 7 - 3(x-4) + 5(x-4)^2 - \frac{2(x-4)^3}{3!} + 6(x-4)^4$

a. $f(4) = 7$

$f'''(4) = -2 \cdot 3! = -12$

b. $P_2(x) \approx f'(x) = -3 + 10(x-4) - 6(x-4)^2$

c. $g(x) = \int_4^x f(t)\,dt$

$7t - \frac{3(t-4)^2}{2} + \frac{5(t-4)^3}{3} - \frac{2(t-4)^4}{4} \Big|_4^x$

$7(x) - \frac{3(x-4)^2}{2} + \frac{5(x-4)^3}{3} - \frac{2(x-4)^4}{4} - 28$

$7(x-4) - \frac{3(x-4)^2}{2} \ldots$

$7x - 28$

$7(x-4)$

20 terms were used to approx. the sum of $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(2n-5)}{3^n}$

What is the error bound for the approx.?

a 3rd order Taylor Poly. was used to approx. e^{2x} on $|x| < .5$

Find the error bound for the approx.

$$|E| \le \frac{16e^{1}(.5)^4}{4!}$$

M ← max value on the (n+1) der. graph

$f = e^{2x}$

$f' = 2e^{2x}$

$f'' = 4e^{2x}$

$f''' = 8e^{2x}$

$f^{4} = 16e^{2x(.5)}$

$$\sum_{n=1}^{\infty} \frac{5}{n^2} = 5\sum_{n=1}^{\infty} \frac{1}{n^2}$$

$$\sum_{n=1}^{\infty} \frac{\frac{n^2+5n}{n^3+7n-5}}{\frac{1}{\cancel{n}}} \cdot \frac{n}{1} = 1 \quad \#$$

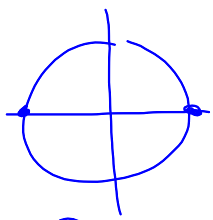
$$x e^{-x^2}$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

$$x \cdot e^{-x^2} = x\left(1 + (-x^2) + \frac{(-x^2)^2}{2} + \frac{(-x^2)^3}{3!} + \cdots\right)$$

$$\sum_{n=0}^{\infty} \frac{x\left((-x^2)^n\right)}{n!}$$

$$\sum_{n=1}^{\infty} \frac{\cos n\pi}{n}$$



$\cos 1\pi \quad \cos 2\pi \quad \cos 3\pi$

$$\frac{-1}{1} + \frac{1}{2} - \frac{1}{3} + \cdots$$

$$\sum_{n=1}^{\infty} \left|\frac{(-1)^n}{n}\right|$$

$$\sum_{n=1}^{\infty} \frac{1}{n^3+2}$$

$$\frac{1}{n^3+2} < \underline{\underline{\frac{1}{n^3}}}$$

$p = 3 > 1$ Con.

$$\int x^2$$

$$\int (x+4)^3$$

$$\frac{(x+4)^4}{4}$$

8.

e^{5x} $n=4$ $x=2$

$$\frac{e^{10}}{} + 5e^{10}(x-2) + \frac{25e^{10}(x-2)^2}{2!} + \frac{(x-2)^3}{3!} + \frac{(x-2)^4}{4!}$$

$f(2) = e^{10}$

$f'(2) = 5e^{5x}$

$f''(2) = 25e^{5x}$

$$\sum_{n=2}^{\infty} \frac{1}{\sqrt{n}-1}$$

$$\frac{1}{\sqrt{n}-1} > \frac{1}{\sqrt{n}}$$

p-series

$p = \frac{1}{2}$ diverges