8.

$$\frac{e^{10}}{} + 5e^{10}(x-2) + \frac{25e^{10}(x-2)^2}{2!} + \frac{(x-2)^3}{3!}$$

$$+ \frac{(x-2)^4}{4!}$$

13.

$$\sum_{n=1}^{\infty} n \sin\left(\frac{1}{n}\right)$$

$$\infty \cdot 0$$

$$\lim_{n\to\infty} \frac{\sin\left(\frac{1}{n}\right)}{\frac{1}{n}} = \lim_{n\to\infty} \frac{\cos\left(\frac{1}{n}\right)\cancel{\frac{-1}{n^2}}}{\cancel{\frac{-1}{n^2}}} = 1$$

$\therefore \sum_{n=1}^{\infty} n \sin\left(\frac{1}{n}\right)$ Diverges by n^{th} term test

$\lim_{n\to\infty} a_n \neq 0$ Div.

20.

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}\sqrt{n}+1}{n+1}$$

converges by A.S.T.

1. alt ✓
2. dec. in mag. ✓
3. $a_n \to 0$ ✓

22.

$$\sum_{n=1}^{\infty} \left(\frac{n}{n^2} - \frac{1}{n^2} \right)$$

$$\lim_{n\to\infty} \frac{\frac{n-1}{n^2} \cdot \frac{n}{1}}{\cancel{\frac{1}{n}}} = 1$$

grow @ same rate

$\frac{1}{n}$ diverges then $\frac{1}{n} - \frac{1}{n^2}$ also diverges

16.

$$\sum_{n=1}^{\infty} \frac{\frac{5n^3 - 3n}{n^2(n+2)(n^2+5)}}{\cancel{\frac{1}{n^2}}} \cdot \frac{n^2}{1} = 5$$

grow @ same rate

$\frac{1}{n^2}$ converges by P-series $p=2$

$$\sum_{n=1}^{\infty} \frac{5}{n^2} = 5 \sum_{n=1}^{\infty} \frac{1}{n^2}$$

23.

$$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1+n}{n^2}$$

$$|E| < \frac{101}{100^2}$$

Absolutely convergent ??

$$\sum_{n=1}^{\infty} \frac{\frac{1+n}{n^2}}{\frac{1}{n}}$$ Diverges

$$\frac{1}{n^1} \quad p=1$$

49.

$$\sum_{n=1}^{\infty} \frac{(x+\pi)^n}{\sqrt{n}}$$

$$\frac{(x+\pi)^{\cancel{n}+1}}{(n+1)^{\frac{1}{2}}} \cdot \frac{n^{\frac{1}{2}}}{\cancel{(x+\pi)^n}}$$

$$\lim_{n\to\infty} \frac{n^{\frac{1}{2}}(x+\pi)}{(n+1)^{\frac{1}{2}}} = x+\pi$$

$$-1 < x+\pi < 1$$

$$-1-\pi \le x < 1-\pi$$

$$\sum_{n=1}^{\infty} \frac{(-1-\cancel{\pi}+\cancel{\pi})^n}{\sqrt{n}}$$

$$\sum_{n=1}^{\infty} \left| \frac{(-1)^n}{n^{\frac{1}{2}}} \right|$$

alt ✓
dec in mag. ✓
$a_n \to 0$ ✓

Converges

$$\sum_{n=1}^{\infty} \frac{(1-\pi+\pi)^n}{\sqrt{n}}$$

$$\sum_{n=1}^{\infty} \frac{1^n}{n^{\frac{1}{2}}} = \frac{1}{n^{\frac{1}{2}}}$$

$p = \frac{1}{2}$

Diverges

23.

25.

Error:

✗ Alt Series Error
Truncation Error

$|E| <$ next term

✗ Taylor Remainder
Lagrange Error

$$|E| \leq \frac{M (x-a)^{n+1}}{(n+1)!}$$

M: max value of $f^{n+1}(x)$ on the interval

Intervals of Convergence

1. Ratio $n+1$ — check endpts. — <1 Con. $=1$ find new test >1 Div.
2. Geometric $(\ )^n$ $|r|<1$ Con. — no endpts ever
3. P-series $p>1$ Con. $p \leq 1$ div.
4. n^{th} term for divergence $\lim a_n \neq 0$ Div.
5. A.S.T. Con. $\begin{cases} \text{alt.} \\ \text{dec in mag.} \\ a_n \to 0 \end{cases}$
6. Limit Comparison $\lim\limits_{n\to\infty} \frac{a_n}{b_n} = \#$ — grow @ same rate — I pick b_n — then determine b_n
7. Direct Comparison

$a_n < b_n$ ← I pick

b_n converges then a_n converges

a_n diverges then b_n diverges

8. Integral

$\sum_{n=1}^{\infty} a_n$ then find $\int_1^{\infty} a_n \, dn$

they behave the same