Warm-Up

Ex. (a) Find the third-degree Maclaurin polynomial for $f(x) = e^x$.

$$1 + x + \frac{x^2}{2!} + \frac{x^3}{3!}$$

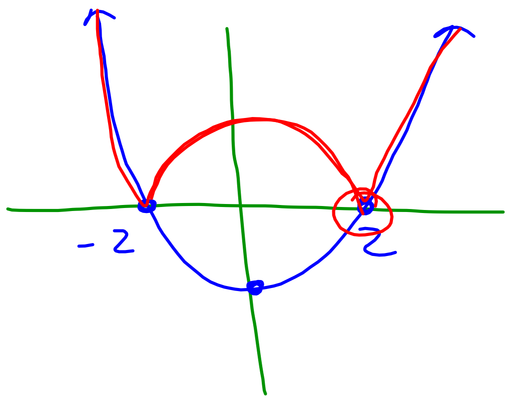
(b) Use your answer to (a) to find:

$$\lim_{x\to 0} \frac{f(x)-1}{2x}$$

$$\frac{\left(\cancel{1} + x + \frac{x^2}{2!} + \frac{x^3}{3!}\right) - \cancel{1}}{2x}$$

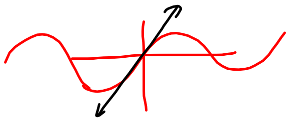
$$\lim_{x\to 0} \frac{1}{2} + \frac{\cancel{x}}{2\cdot 2!} + \frac{\cancel{x^2}}{\cancel{2\cdot 3!}} = \frac{1}{2}$$

7. $|x^2 - 4|$ $a = 2$



9.3

21.



$\sin x \approx x$

$|x| < 10^{-3}$

$$|E| \le \frac{M(x-0)^3}{3!}$$

$$|E| \le \frac{1(.001)^3}{3!}$$

$$|E| \le 1.66\overline{7} \times 10^{-10}$$

9.4b Radius and Interval of Convergence

For what values of x is $\frac{1}{1+x^2} = 1 - x^2 + x^4 - x^6 + \ldots + (-1)^n x^{2n} + \ldots$

Convergent?

$\frac{a_1}{1-(r)}$

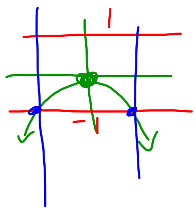
Analytically:

Graphically:

$r = -x^2$

$-1 < -x^2 < 1$

$|-x^2| < 1$

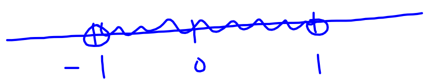


$-1 < x < 1$

$|x| < 1$

$R = 1$

What are the interval and radius of convergence?



Find the radius of convergence

$$\sum_{n=0}^{\infty} \frac{nx^n}{10^n}$$

$$\lim_{n\to\infty} \left| \frac{(n+1)\,x^{\cancel{n+1}}}{10^{\cancel{n+1}}} \cdot \frac{\cancel{10^n}}{n\,\cancel{x^n}} \right|$$

$$\lim_{n\to\infty} \left| \frac{\cancel{(n+1)}^{1}}{n} \frac{x}{10} \right| = \left| \frac{x}{10} \right|$$

$$\cancel{10} \cdot \left| \frac{x}{\cancel{10}} \right| < 1 \cdot 10$$

Intervals $\rightarrow |x| < 10$ Radius: 10

$-10 < x < 10$

Find the radius of convergence

$$\sum_{n=\cancel{0}}^{\infty} n!\,x^n = 1 \cdot 0 + 2!\,0^2 + 3! \cdot 0^3$$

$$\frac{(n+1)\cancel{n!}\; x^{\cancel{n}+1}}{\cancel{n!}\; \cancel{x^n}}$$

$$\cancel{(n+1)!}$$

$$\lim_{n\to\infty} (n+1)x = \infty$$

$x = 0$

0

$$\lim_{n \to \infty} \frac{x}{n+1} = 0$$

$$\sum_{n=0}^{\infty} (-1)^n (3x+5)^n$$

$r = -1(3x+5)$

$|-1(3x+5)| < 1$ $\quad |3x-5|$

$\cancel{|-1|} \cdot |3x+5| < 1$

$|3x+5| < 1$ $\quad \left|3\left(x+\frac{5}{3}\right)\right| < 1$

$-1 < 3x+5 < 1$ $\quad \left|x+\frac{5}{3}\right| < \frac{1}{3}$

$-5 \qquad\qquad -5$

$\frac{-6}{3} < \frac{3x}{3} < \frac{-4}{3}$

$-\frac{6}{3} < x < -\frac{4}{3}$

$R = \frac{1}{3}$

$$|3x+5| < 1$$
$$\quad -5 \quad -5$$
$$|3x| < -4$$

Find the radius of convergence

$$\sum_{n=0}^{\infty} \frac{\sqrt{n}x^n}{3^n}$$

$$\frac{\sqrt{n+1}\; x^{\cancel{n}+1}}{3^{\cancel{n}+1}} \cdot \frac{\cancel{3^n}}{\sqrt{n}\; \cancel{x^n}}$$

$$\lim_{n\to\infty} \frac{x\sqrt{n+1}}{3\sqrt{n}} = \left|\frac{x}{3}\right| < 1$$

$$3 \cdot \left|\frac{x}{3}\right| < 1 \cdot 3$$

$$|x| < 3$$

$$-3 < x < 3$$

Radius $= 3$