Warm-Up

(a) Find the Maclaurin polynomial of degree $n = 5$ for $f(x) = \sin x$

$P_5 = x - \frac{x^3}{3!} + \frac{x^5}{5!}$

(b) Find $P_5(1.2)$ .9327

What is the value of $f(1.2)$ .93203

What is the error of your approximation? .000697

The error is symbolized $|f(1.2) - P_5(1.2)|$

(c) Find $P_5(2.1)$ .896842

What is the value of $f(2.1)$ .863209

$|f(2.1) - P_5(2.1)| =$ .03369

How does the error for $P_5(2.1)$ compare to the error for $P_5(1.2)$

What do you think would happen if we used our polynomial to estimate sin 2.7?

9.4a Tests for Convergence of Series

Geometric Series Test

A geometric series is in the form $\sum_{n=0}^{\infty} a_1 r^n$ or $\sum_{n=1}^{\infty} a_1 r^{n-1}$, $a \neq 0$.

The geometric series **diverges** if $|r| \geq 1$

If $|r| < 1$, the series **converges** to the sum $S = \frac{a_1}{1-r}$.

Determine if the following series converges or diverges.

$$\sum_{n=1}^{\infty} \frac{3}{2^n} = 3(2^{-n}) = 3(2^{-1})^n \quad r = \frac{1}{2} < 1$$

$3\left(\frac{1}{2^n}\right)$ $\quad S = \frac{\frac{3}{2}}{1-\frac{1}{2}} = \frac{\frac{3}{2}}{\frac{1}{2}} = 3$ Converges

$3\left(\frac{1}{2}\right)^n$

$$\sum_{n=1}^{\infty} \left(\frac{3}{2}\right)^n \quad r = \frac{3}{2} > 1 \text{ Diverges}$$

nth Term Test for Divergence

If $\boxed{\lim_{n\to\infty} a_n \neq 0}$, then the series $\sum_{n=1}^{\infty} a_n$ diverges.

read note on notes page!!

Determine if the following series converges or diverges.

(a) $\sum_{n=1}^{\infty} \frac{2n+3}{3n-5}$ $a_n \to \frac{2}{3}$ Diverges

(b) $\sum_{n=1}^{\infty} \frac{n!/n!}{2n!/n! + \cancel{1}/n!}$ $a_n \to \frac{1}{2}$ Diverges

(c) $\sum_{n=1}^{\infty} \frac{3^n/3^n - \cancel{2}/3^n}{3^n/3^n}$ $a_n \to 1$ Diverges

d. $\sum_{n=1}^{\infty} \frac{1}{n^3+1}$ $a_n \to 0$?

Ratio Test

Let $\sum_{n=1}^{\infty} a_n$ be a series of nonzero terms and $\boxed{\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = L}$

1) $\sum_{n=1}^{\infty} a_n$ converges if L< 1.

2) $\sum_{n=1}^{\infty} a_n$ diverges if L> 1.

3) If L = 1 the test is inconclusive. ?

(a) $\sum_{n=1}^{\infty} \frac{2^n}{n!}$

$$\frac{\frac{2^{n+1}}{(n+1)!}}{\frac{2^n}{n!}} \qquad \frac{2^{n+1}}{(n+1)!}\cdot\frac{n!}{2^n} = \frac{\cancel{2^n}\cdot 2^1}{(n+1)(\cancel{n!})}\cdot\frac{\cancel{n!}}{\cancel{2^n}}$$

$$\lim_{n\to\infty} \frac{2}{n+1} = 0 < 1$$

converge

$\sum_{n=1}^{\infty} \frac{n^2 3^{n+1}}{2^n}$

$$\frac{(n+1)^2 3^{(n+1)+1}}{2^{n+1}}\cdot\frac{2^n}{n^2 3^{n+1}}$$

$$\frac{(n+1)^2\, \cancel{3^{n+1}}\cdot 3^1}{\cancel{2^n}\cdot 2}\cdot\frac{\cancel{2^n}}{n^2\cdot \cancel{3^{n+1}}} = \frac{3(n+1)^2}{2n^2}$$

$$\lim_{n\to\infty} \frac{3}{2}\boxed{\frac{(n+1)^2}{n^2}} = \frac{3}{2} > 1$$

Diverges

$\sum_{n=1}^{\infty} \frac{(n+1)!}{3^n}$

$$\frac{((n+1)+1)!}{3^{n+1}}\cdot\frac{3^n}{(n+1)!}$$

$$\frac{(n+2)!}{3^{n+1}}\cdot\frac{3^n}{(n+1)!}$$

$$\frac{(n+2)\cancel{(n+1)!}}{3^{\cancel{n}+1}}\cdot\frac{\cancel{3^n}}{\cancel{(n+1)!}} = \frac{n+2}{3}$$

$$\lim_{n\to\infty} \frac{n+2}{3} = \infty$$

new test

Diverges