On what interval, does the 3rd order Maclaurin series approximate $\sin x$ within .01?

$$\left|\sin x - \left(x - \frac{x^3}{3!}\right)\right| \leq .01$$

$$-1.042 \leq x \leq 1.04251$$

Taylor's Remainder Estimation Theorem

with an nth order polynomial

$$f(x) \approx f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \ldots + \frac{f^{n}(a)}{n!}(x-a)^n$$

$$|\text{Error}| \leq M\frac{|(x-a)|^{n+1}}{(n+1)!}$$

M is the max value of $f^{n+1}(x)$ on the interval

The approximation $\ln(1+x) \approx x - \frac{x^2}{2}$ is used when x is small.

$x = 0$

Use the Remainder Estimation Theorem to get a bound for the maximum error when $|x| \le .01$ $\quad -.01 \le x \le .01$

Support your answer graphically.

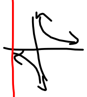
$$|E| \le \frac{\overset{2.061}{M}\,(\overset{.01}{x-0})^3}{3!}$$

$f(x) = \ln(1+x)$

$f'(x) = \frac{1}{1+x}$

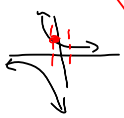
$f''(x) = \frac{-1}{(1+x)^2}$

$f'''(x) = \frac{2}{(1+x)^3}$ $\qquad \frac{1}{x}$



$$|E| \le \frac{2.061(.01)^3}{3!}$$

$$|E| \le 3.435 \times 10^{-7}$$



$$\frac{2}{(1+(-.01))^3} = 2.061$$

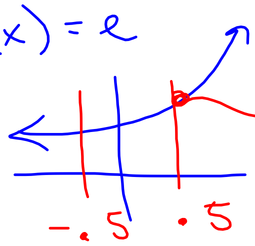
Give an error bound when e^x is approximated by the 4th degree polynomial about x = 0 for $|x| \le .5$

$f(x) = e^x$

$\vdots$

$f^5(x) = e^x$

$$|E| \le \frac{m\,(x)^5}{5!}$$



$e^{.5}$

$$|E| \le \frac{e^{.5}(.5)^5}{5!}$$

$$|E| \le 4.2935 \times 10^{-4}$$

$$-4.2935 \times 10^{-4} \le E \le 4.2935 \times 10^{-4}$$

What values of x may be used in $1-\frac{x^2}{2!}+\frac{x^4}{4!}$ to approximate $\cos x$ with an error no greater than 5×10^{-4}

$$|E| \le \frac{M(x)^6}{6!}$$

$$6!(.0005) \le 1\frac{(x)^6}{\cancel{6!}}$$

$$\sqrt[6]{.36} = \sqrt[6]{x^6}$$

$$.8434 = x$$

$$-.8434 \le x \le .8434$$

$$|x| \le .8434$$

5. $(1-x)^{-2}$ $x=0$

$$\boxed{1 + \overset{2!}{2}(x) + \frac{\overset{3!}{6}(x)^2}{2!} + \frac{\overset{4!}{24}x^3}{3!} + \frac{5!\,x^4}{4!}}$$

$$f'(x) = -2(1-x)^{-3}(-1)$$

$$2(1-x)^{-3}\Big|_{x=0}$$

$1+2x+3x^2+4x^3+5x^4$

$$f''(x) = -6(1-x)^{-4}(-1)$$

$$6(1-x)^{-4}\Big|_{x=0} = 6$$

$$f'''(x) = 24(1-x)^{-5}\Big|_{x=0} = 24$$

19. $\sin x \approx x - \frac{x^3}{6} + \frac{0x^4}{4!} + \frac{x^5}{5!}$ $|E| \leq .0005$

$$\sqrt[4]{4!(.0005)} \leq \sqrt[4]{\frac{1(x)^{\cancel{4}5}}{\cancel{4!}\,5!}}$$

$-.3309 \leq x \leq .3309$

$|x| \leq .3309$

$|x| \leq .5969$

21. $\sin x \approx \boxed{x + \frac{0x^2}{2!}} + \frac{x^3}{3!}$ $|x| \leq .001$

$$|E| < \frac{1(x)^3}{3!}$$