Warm-Up

Ex. (a) Find the third-degree Maclaurin polynomial for $f(x) = e^x$.

$$e^x \approx \left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!}\right)$$

(b) Use your answer to (a) to find: $\lim_{x\to 0} \frac{f(x) - 1}{2x}$

$$\lim_{x\to 0} \frac{\left(\cancel{1} + x + \frac{x^2}{2} + \frac{x^3}{3!}\right) - \cancel{1}}{2x}$$

$$\lim_{x\to 0} \left(\frac{1 + \cancel{\frac{x}{2}}^{0} + \cancel{\frac{x^2}{3!}}^{0}}{2}\right) = \frac{1}{2}$$

9.2b

25c. $g(x) = \frac{e^x - 1}{x} = 1 + \frac{x}{2} + \frac{x^2}{3!} + \frac{x^3}{4!} \cdots$

$$g'(x) = \frac{x(e^x) - (e^x - 1)\cdot 1}{x^2} = \frac{1}{2} + \frac{2x}{3!} + \frac{3x^2}{4!} + \cdots$$

$$\frac{xe^x - e^x + 1}{x^2} = \sum_{n=1}^{\infty} \frac{n x^{n-1}}{(n+1)!}$$

$$\frac{e^1 - e^1 + 1}{1^2} = \sum_{n=1}^{\infty} \frac{n}{(n+1)!}$$

$$1 = \sum_{n=1}^{\infty} \frac{n}{(n+1)!}$$

26.

$f(x) = \dfrac{2}{1-t^2}$ $\qquad G(x) = \int_0^x f(t)\,dt$

Geometric

$r = t^2$

$2 + 2t^2 + 2t^4 + 2t^6 + \cdots + 2t^{2n} + \cdots$

$= \sum_{n=0}^{\infty} 2t^{2n}$

$\left(2t + \dfrac{2t^3}{3} + \dfrac{2t^5}{5} + \dfrac{2t^7}{7} + \cdots \right)\Big|_0^x$

$G(x) = 2x + \dfrac{2x^3}{3} + \dfrac{2x^5}{5} + \dfrac{2x^7}{7} + \cdots$

$\sum_{n=1}^{\infty} \dfrac{2x^{2n-1}}{(2n-1)}$

33. $(x-2)^3$

$\ln x$ @ $x=2$

$\dfrac{f'''(2)(x-2)^3}{3!}$

$f(x) = \ln x$

$f'(x) = \dfrac{1}{x}$

$f''(x) = -\dfrac{1}{x^2}$

$f'''(x) = \dfrac{2}{x^3}\Big|_{x=2} = \dfrac{1}{4}$

$\dfrac{\frac{1}{4}(x-2)^3}{3!} = \dfrac{1}{24}(x-2)^3$

9.3 Taylor Series with Remainder

What is the 5th order Maclaurin series for $f(x) = \sin x$

What is the maximum error when approximating $\sin x$ on $[-\pi, \pi]$

$|E| \leq .5240$

Solve Graphically and Numerically:

$$\sin x \approx x - \frac{x^3}{3!} + \frac{x^5}{5!}$$

$$E = \text{actual} - \text{approx.}$$

$$|E| = \sin \cancel{x}\pi - \left(\cancel{x}\pi - \frac{\cancel{x}\pi^3}{3!} + \frac{\cancel{x}\pi^5}{5!}\right)$$

$$|E| = .5240$$

How many terms are needed in the Maclaurin series for $\sin x$ in order to approximate $\sin x$ within .0001 on $[-\pi, \pi]$

On what interval, does the 3rd order Maclaurin series approximate $\sin x$ within .01?

Taylor's Remainder Estimation Theorem

with an nth order polynomial

$$f(x) \approx f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \ldots + \frac{f^n(a)}{n!}(x-a)^n$$

$$\text{Error} \leq \left| M \frac{(x-a)^{n+1}}{(n+1)!} \right|$$

M is the max value of $f^{n+1}(x)$ on the interval

The approximation $\ln(1+x) \approx x - \frac{x^2}{2}$ is used when x is small.

Use the Remainder Estimation Theorem to get a bound for the maximum error when $|x| \leq .01$

Support your answer graphically.

Give an error bound when e^x is approximated by the 4th degree polynomial about x = 0 for $x \leq .5$

What values of x may be used in $1-\frac{x^2}{2!}+\frac{x^4}{4!}$ to approximate $\cos x$ with an error no greater than 5×10^{-4}