Warm-Up

Ex. Suppose that the function $f(x)$ is approximated near $x = 0$ by a third-degree Taylor polynomial $P_3(x) = 2 - 5x^2 + 8x^3$

$8(3x^2$

$3 \cdot 2x^1$

$8(3 \cdot 2 \cdot 1)$

(a) Find the value of $f(0)$, $f'(0)$, $f''(0)$, and $f'''(0)$

$P(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3$

(b) Does f have a local maximum, a local minimum, or neither at $x = 0$? Justify your answer.

$f(0) = 2$ $f'(0) = 0$ $\frac{f''(0)}{2!} = -5 \cdot 2 = -10$

$\frac{f'''(0)}{3!} = 8 \cdot 6 = 48$

$f(x)$ has a max @ $x = 0$ because $f'(0) = 0$ and $f''(0) = -10$ $\therefore$ concave down meaning a max @ $x = 0$

QR: #3

$f = 3^x$

$f' = 3^x \ln 3 \longrightarrow \ln 3(3^x)$

$f'' = \ln 3(3^x \ln 3) = 3^x(\ln 3)^2$

$f''' = 3^x(\ln 3)^3$

$f^n = 3^x(\ln 3)^n$

23. $f(1)=4 \quad f'(1)=-1 \quad f''(1)=3 \quad f'''(1)=2$

a. $P_3 = 4 - 1\underline{(x-1)} + \frac{3}{2!}(x-1)^2 + \frac{2}{3!}(x-1)^3$

$P(1.2) = 3.863$

b. $P' = -1 + \frac{3}{\cancel{2}}\left(\cancel{2}(x-1)^1\right)\cdot 1 + \frac{\cancel{2}}{\cancel{3!}}\left(\cancel{3}(x-1)^2\right)$

$P' = -1 + 3(x-1) + (x-1)^2$

$P'(1.2) = -.36$

15. a. $x=0$ b. $x=1$

$f(x) = x^3 - 2x + 4\,\big|_{x=1}$

$f'(x) = 3x^2 - 2\,\big|_{x=1}$

$f''(x) = 6x\,\big|_{x=1}$

$f'''(x) = 6$

a. $P_3 = 4 + \frac{-2}{1!}x + \frac{0}{2!}x^2 + \frac{6}{3!}x^3$

b. $P_3 = 3 + 1(x-1) + \frac{6}{2!}(x-1)^2 + \frac{6}{3!}(x-1)^3$

41. $f(x) = \sin x$ @ $x = \frac{\pi}{2}$

$f'(x) = \cos x$

$f''(x) = -\sin x$

$$1 + 0 + \frac{-1}{2}\left(x - \frac{\pi}{2}\right)^2$$

Use a known Maclaurin series to write a series for:

$$x\sin x = x\left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots\right)$$

$$x\sin x = x^2 - \frac{x^4}{3!} + \frac{x^6}{5!} - \frac{x^8}{7!} + \cdots = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}(x)^{2n}}{(2n-1)!}$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots$$

$x = x^2$

$$\cos x^2 = 1 - \frac{(x^2)^2}{2!} + \frac{(x^2)^4}{4!} - \frac{(x^2)^6}{6!} + \cdots$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n (x^2)^{2n}}{(2n)!}$$

$$\sum_{n=0}^{\infty} \frac{(-1)^n (x)^{4n}}{(2n)!}$$

$0! = 1$ by definition

$$e^x - 1 = \cancel{1} + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots \cancel{-1}$$

$$\frac{e^x - 1}{2} = \frac{x}{2} + \frac{x^2}{2 \cdot 2!} + \frac{x^3}{2 \cdot 3!} + \frac{x^4}{2 \cdot 4!} + \cdots$$

$$\frac{e^x - 1}{2} = \sum_{n=1}^{\infty} \frac{x^n}{2 \cdot n!}$$

$$\frac{e^x}{2} - \frac{1}{2}$$

5.

$\sin 2x$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$$

$$\sin 2x = \frac{(2x)^1}{1!} - \frac{(2x)^3}{3!} + \frac{(2x)^5}{5!} + \cdots \frac{(-1)^{n-1}(2x)^{2n-1}}{(2n-1)!}$$

$$= \sum_{n=1}^{\infty} \frac{(-1)^{n-1}(2x)^{2n-1}}{(2n-1)!}$$