17. $\sum_{n=0}^{\infty} \sin^n\left(\frac{\pi}{4} + n\pi\right)$

$a_1 = 1$

$r = -\frac{\sqrt{2}}{2}$ ✓

$\sin^0\left(\frac{\pi}{4}\right) = 1$

$\sin^1\left(\frac{5\pi}{4}\right) = -\frac{\sqrt{2}}{2}$

$S = \frac{1}{\frac{2}{2} + \left(+\frac{\sqrt{2}}{2}\right)}$

$\sin^2\left(\frac{9\pi}{4}\right) = \left(\frac{\sqrt{2}}{2}\right)^2$

$\frac{1}{\frac{2+\sqrt{2}}{2}}$

$\sin^3\left(\frac{13\pi}{4}\right) = \left(-\frac{\sqrt{2}}{2}\right)^3$

$S = \frac{2}{2+\sqrt{2}}$

23 $\sum_{n=0}^{\infty} \left(-\frac{1}{2}\right)^n (x-3)^n = 1 + -\frac{1}{2}(x-3) + \left(\frac{1}{2}\right)^2(x-3)^2 + \cdots$

$\sum_{n=0}^{\infty} \left(-\frac{1}{2}(x-3)\right)^n$

$r = -\frac{1}{2}(x-3)$

$S = \frac{1}{1-\left(-\frac{1}{2}(x-3)\right)}$

$-1 < \cancel{-\frac{1}{2}}(x-3) < 1$

$\cdot -2 \quad \cdot \cancel{-2} \quad \cdot -2$

$2 > x-3 > -2$

$+3 \quad +3 \quad +3$

$5 > x > 1$

$1 < x < 5$

63.

$$\frac{1}{x} = \frac{1}{1-1+x} = \frac{1}{1-(1-x)}$$

$$\int \frac{1}{x} = \int \frac{1}{1-(1-x)} = \int \left(1 + (1-x) + (1-x)^2 + (1-x)^3 + \cdots\right)$$

$$\ln x = x + \frac{-(1-x)^2}{2} + \frac{-(1-x)^3}{3} + \frac{-(1-x)^4}{4}$$

$$\ln x = x - \frac{(\cancel{x}(x-1))^2}{2} - \frac{((-1)\cancel{x}(x-1))^3}{3}$$

$$\ln x = x - \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3} - \frac{(x-1)^4}{4} + \cdots$$

$$\ln x = (-1)^{n+1}(x) + \frac{(-1)^{n+1}(x-1)^2}{2} + \frac{(-1)^{n+1}(x-1)^3}{3} + \cdots$$

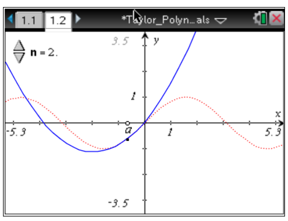
$a_1 = 3 \qquad r = x+1$

$3 + 3(x+1) + 3(x+1)^2$

9.2a Taylor Series

What is the easiest family to integrate and take derivatives of?

How can we find a polynomial that looks like another function?



$x - \frac{1}{6}x^3$

power series: $p(x)$

$$p(x) = c_0 + c_1 x + c_2 x^2 + c_3 x^3 + \ldots$$

$0 + 1x + \frac{0}{2}x^2 - \frac{1}{6}x^3 +$

$$p(x) \approx f(x)$$

we have to find c's - coefficients

$$* \ \sin x = c_0 + c_1 x + c_2 x^2 + c_3 x^3 + c_4 x^4 + \cdots$$
$$\sin 0 = c_0 + 0 + 0 + 0 \cdots$$
$$0 = c_0$$

$$* \ \cos x = c_1 + 2c_2 x + 3c_3 x^2 + 4c_4 x^3 + \cdots$$
$$\cos 0 = c_1 + 0 + 0 \cdots$$
$$1 = c_1$$

$$* \ -\sin x = 2c_2 + 6c_3 x + 12c_4 x^2 + 20c_5 x^3$$
$$-\sin 0 = 2c_2$$
$$0 = 2c_2$$

$$-\cos x = 6c_3 + 24c_4 x + 60c_5 x^2 \cdots$$

$$-\cos 0 = 6c_3$$
$$\frac{-1}{6} = \frac{6c_3}{6}$$
$$-\frac{1}{6} = c_3$$

$$\sin x = \frac{x^1}{1!} - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$$

Find a parabola that fits $y = e^x$ for x near 0.

$e^x \approx 1 + 1x + \frac{1}{2}x^2$

$e^x = c_0 + c_1 x^1 + c_2 x^2 + c_3 x^3 + \ldots$

$e^0 = c_0 + 0 \ldots$

$1 = c_0$

$e^x = c_1 + 2c_2 x + 3c_3 x^2$

$e^0 = c_1 + 0$

$e^x = 2c_2 + 6c_3 x$

$e^0 = 2c_2$

$\frac{1}{2} = c_2$

Find an nth degree polynomial that fits $y = e^x$ for x near 0.

$e^x \approx 1 + x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \frac{1}{4!}x^4 + \ldots + \frac{x^n}{n!}$

Construct a 4th degree polynomial that matches

$y = \ln(1+x)$ at $x = 0$

$$\ln(1+x) = C_0 + C_1 x + C_2 x^2 + C_3 x^3 + \cdots$$