Sequence:

1, 3, 5, 7, 9, 11, . . .

Series:

1 + 3 + 5 + 7 + 9 + 11 + . . .

1, 3, 9, 27, 81, . . .

1 + 3 + 9 + 27 + 81 + . . .

9.1 Power Series

Infinite Series $\sum_{k=1}^{\infty} a_k$

$\sum_{k=1}^{\infty} 2k = 2 + 4 + 6 + 8 + \ldots$

to find the sum of an infinite series - we look at the partial sums

Partial Sums

$S_1 = a_1$ $S_1 = 2$

$S_2 = a_1 + a_2$ $S_2 = 6$

$2 + 4$

$S_3 = a_1 + a_2 + a_3$ $S_3 = 12$

$2 + 4 + 6$

$S_n = \sum_{k=1}^{n} a_k$

what if the partial sums have a finite limit?

Series Converges

Does the series converge or diverge?

$$1+1.1+1.11+1.111+\ldots$$

Diverges

$S_1 = 1$

$S_2 = 1 + 1.1 = 2.1$

$S_3 = 1 + 1.1 + 1.11 = 3.21$

$S_4 = 3.21 + 1.111 = 4.321$

$$5+.5+.05+.005+\ldots$$

Converging to:

$5.\overline{5}$

$5\frac{5}{9}$

$S_1 = 5$

$S_2 = 5.5$

$S_3 = 5.55$

$S_4 = 5.555$

Infinite Geometric Series

$$\sum_{n=1}^{\infty} ar^{n-1} = a + ar + ar^2 + ar^3 + \ldots$$

When do geometric series converge?

$-1 < r < 1$ $\quad |r| < 1$

What is the interval of convergence?

$(-1, 1)$

What is the sum?

$$\sum_{n=1}^{\infty} ar^{n-1} = a + ar + ar^2 + ar^3 + \ldots = \frac{a_1}{1-r}$$

$$S = \frac{4}{1-\left(-\frac{3}{4}\right)} = \frac{4}{\frac{7}{4}} = \frac{16}{7}$$

Tell whether each series converges or diverges. If it converges, give its sum.

$$\sum_{n=1}^{\infty} 3\left(\frac{1}{2}\right)^{n-1}$$ $r=\frac{1}{2}$ $-1<\frac{1}{2}<1$ $S=\frac{3}{1-\frac{1}{2}}=6$

$1-\frac{1}{2}+\frac{1}{4}-\frac{1}{8}+\ldots$ $r=-\frac{1}{2}$ Converges to: $S=\frac{1}{1-\left(-\frac{1}{2}\right)}=\frac{2}{3}$

$$\sum_{k=0}^{\infty}\left(\frac{3}{5}\right)^{k}$$ $r=\frac{3}{5}$ Converges to: $S=\frac{1}{1-\frac{3}{5}}=\frac{5}{2}$

$$\sum_{n=1}^{\infty}\left(\frac{3n-1}{2n+1}\right)$$

$\lim_{n\to\infty}\frac{3n-1}{2n+1}=\frac{3}{2}$ $\therefore$ Diverging

Test for Divergence:

If $\lim_{n\to\infty} a_n \neq 0$ or DNE then the series diverges.

$$\sum_{n=1}^{\infty}\frac{n^2}{5n^2+4}$$ $a_n \to \frac{1}{5}$ Diverging

$$\sum_{n=1}^{\infty}\frac{1}{n(n+4)}$$ $a_n \to 0$ not enough info.

Find the function that is equal to (models) the series. Support graphically.

$$1+x+x^2+x^3+\ldots+x^n+\ldots = \frac{1}{1-x}, \quad |x|<1$$

geometric w/ $r=x$

$$\sum_{n=0}^{\infty} x^n$$

Power Series: series equivalent to a known function on a given interval

centered at $x = 0$: $\displaystyle\sum_{n=0}^{\infty} c_n x^n = c_0 x^0 + c_1 x^1 + \ldots$

centered at $x = a$: $\displaystyle\sum_{n=0}^{\infty} c_n (x-a)^n$

Find the power series for the following functions:

$$f(x) = \frac{1}{1+x} = \frac{1}{1-(-x)} = 1 - x + x^2 - x^3 + \ldots = \sum_{n=0}^{\infty} (-x)^n$$

$$f(x) = \frac{1}{x} = \frac{1}{1+(x-1)} = \frac{1}{1-(-x+1)}$$

hint: $\frac{1}{x} = \frac{1}{1+(x-1)}$

$$1 + (-x+1) + (-x+1)^2 + (-x+1)^3 + \ldots = \sum_{n=0}^{\infty} (-x+1)^n$$

Differentiate the power series for $\frac{1}{1-x}$

What function equals this new power series?

$$(1-x)^{-1} = \frac{1}{1-x} = 1 + x + x^2 + x^3 + \ldots$$

$$-1(1-x)^{-2}(-1)$$

$$\frac{1}{(1-x)^2} = 0 + 1 + 2x^1 + 3x^2 + 4x^3 + \ldots$$

$$\sum_{n=1}^{\infty} n x^{n-1}$$

Use the power series for $\frac{1}{1+x}$ to find a power series for $\ln(1+x)$

$$\frac{1}{1+x} = 1 - x + x^2 - x^3 + \cdots$$