Sit 32

10.
$$\int_{-1}^{8} \frac{dx}{\sqrt[3]{x}}$$

$$\lim_{b\to 0}\int_{-1}^{b} x^{-\frac{1}{3}}\,dx + \lim_{b\to 0}\int_{b}^{8} x^{-\frac{1}{3}}\,dx$$

$$\lim_{b\to 0}\left(\frac{3}{2}x^{\frac{2}{3}}\Big|_{-1}^{b}\right) + \lim_{b\to 0}\left(\frac{3}{2}x^{\frac{2}{3}}\Big|_{b}^{8}\right)$$

$$\cancel{\frac{3}{2}(b)^{\frac{2}{3}}}^{0} - \left(\frac{3}{2}(-1)^{\frac{2}{3}}\right) + \lim_{b\to 0}\left(\frac{3}{2}(8)^{\frac{2}{3}} - \cancel{\frac{3}{2}(b)^{\frac{2}{3}}}^{0}\right)$$

$$-\frac{3}{2} + 6 = \frac{9}{2}$$

6.
$$\int_{0}^{1} \frac{x+1}{\sqrt{x^2+2x}}\,dx$$

$$\lim_{b\to 0}\int_{b}^{1} \frac{x+1}{\sqrt{x^2+2x}}\,dx$$

$u = x^2 + 2x$

$\frac{du}{2x+2} = (2x+2)dx$

$$\lim_{b\to 0}\int u^{-\frac{1}{2}}\cancel{(x+1)}\cdot\frac{du}{\cancel{2x+2}}$$
$$2\cancel{(x+1)}$$

$$\frac{1}{2}\lim_{b\to 0}\int u^{-\frac{1}{2}}\,du$$

$$2u^{\frac{1}{2}}$$

$$\frac{1}{\cancel{2}}\left(\lim_{b\to 0}\ \cancel{2}\sqrt{x^2+2x}\Big|_{b}^{1}\right)$$

$$\lim_{b\to 0}\left(\sqrt{1^2+2(1)} - \cancel{\sqrt{b^2+2b}}^{0}\right) = \sqrt{3}$$

5. $$\int_1^4 \frac{dx}{1-x}$$

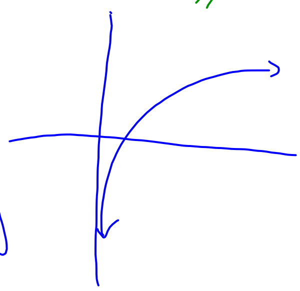
$$\lim_{b\to 1} \int_b^4 \frac{1}{1-x}\,dx$$

$$\lim_{b\to 1} \left(-\ln|1-x| \Big|_b^4 \right)$$

$$\lim_{b\to 1} \left(-\ln 3 - \left(-\ln|1-b| \right) \right)$$

$$-\ln 3 + -\infty$$

Diverging



7. $$\int_{-\infty}^0 \frac{dx}{(2x-1)^3}$$

$$\lim_{b\to -\infty} \int_b^0 (2x-1)^{-3}\,dx$$

$$\lim_{b\to -\infty} \left(\frac{(2x-1)^{-2}}{-2\cdot 2} \Big|_b^0 \right)$$

$$\lim_{b\to -\infty} \left(\frac{1}{-4(2x-1)^2} \Big|_b^0 \right)$$

$$\lim_{b\to -\infty} \left(\frac{1}{-4(2(0)-1)^2} - \left(\frac{1}{-4(2b-1)^2} \right) \right) \quad 0$$

$$\frac{-1}{4} + 0 = -\frac{1}{4}$$

Sat 17

10. $\lim_{x\to0^+} \frac{x}{\ln(x+1)} = \lim_{x\to0^+} \frac{1}{\frac{1}{x+1}}$

$\lim_{x\to0^+} x+1 = 1$

5. $\lim_{x\to0} \frac{\tan x - x}{\sin x - x} = \lim_{x\to0} \frac{\sec^2 x - 1}{\cos x - 1}$

$\frac{2\sec x(\sec x \tan x)}{-\sin x}$

$\lim_{x\to0} \frac{2\,\frac{1}{\cos x}\cdot\frac{1}{\cos x}\cdot\frac{\cancel{\sin x}}{\cos x}}{-\cancel{\sin x}}$

$\lim_{x\to0} \frac{-2}{\cos^3 x} = -2$

39. $$\int_0^\infty \frac{16\tan^{-1}x}{1+x^2}\,dx$$

$$\lim_{b\to\infty}\int_0^b \frac{16\tan^{-1}x}{1+x^2}\,dx \qquad u=\tan^{-1}x \qquad (1+x^2)du=\frac{1}{1+x^2}dx$$

$$16\int u\,du$$

$$8u^2$$

$$\lim_{b\to\infty}\left(8(\tan^{-1}x)^2\Big|_0^b\right)$$

$$\lim_{b\to\infty}\left(8(\tan^{-1}b)^2-8(\tan^{-1}0)^2\right)$$

$$8\left(\frac{\pi^2}{4}\right)-0$$

$$2\pi^2$$

37. $$\int_0^\infty \frac{ds}{(1+s)\sqrt{s}}$$

$$u^2=s \qquad (u)^2=(\sqrt{s})^2$$

$$2\sqrt{s}\cdot du=\frac{1}{\cancel{2}}\,s^{\cancel{-}\frac{1}{2}}ds$$

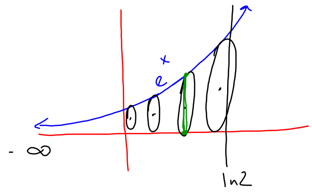
$$\int\frac{2\cancel{\sqrt{s}}\,du}{(1+u^2)\cancel{\sqrt{s}}}$$

$$\int\frac{2}{1+u^2}\,du$$

$$\lim_{b\to\infty}\left(2\tan^{-1}(\sqrt{s})\Big|_0^b\right)$$

$$\lim_{b\to\infty}\left(2\tan^{-1}\sqrt{b}-2\tan^{-1}\sqrt{0}\right)$$

$$2\left(\frac{\pi}{2}\right)-0=\pi$$



πr^2

a. $A = \pi\left(\frac{e^x}{2}\right)^2 = \frac{e^{2x}\pi}{4}$

b. $V = \frac{\pi}{4}\int_{-\infty}^{\ln 2} e^{2x}\,dx$

c. $\frac{\pi}{4}\lim_{b\to-\infty}\int_b^{\ln 2} e^{2x}\,dx$

$\frac{\pi}{4}\lim_{b\to-\infty}\left(\frac{1}{2}e^{2x}\Big|_b^{\ln 2}\right)$

$\frac{\pi}{4}\lim_{b\to-\infty}\left(\frac{1}{2}e^{2(\ln 2)} - \cancel{\frac{1}{2}e^{2b}}\right)^0$

$\frac{\pi}{4}\left(2 - 0\right) = \frac{\pi}{2}$

Sct 17.

5. $\lim_{x\to 0}\frac{\tan x - x}{\sin x - x}$

$\lim_{x\to 0}\frac{\sec^2 x - 1}{\cos x - 1}$

$\lim_{x\to 0}\frac{\tan^2 x}{\cos x - 1}$

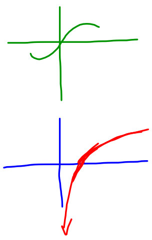
$\lim_{x\to 0}\frac{2\tan x\sec^2 x}{-\sin x}$

$\lim_{x\to 0}\frac{2\,\frac{\cancel{\sin x}}{\cos x}\cdot\frac{1}{\cos^2 x}}{-\cancel{\sin x}}$

$\lim_{x\to 0}\frac{-2}{\cos^3 x} = -2$

8.

$$\lim_{x \to 0^+} \frac{\ln(\sin x)}{\ln(\tan x)}$$



$$\frac{\frac{1}{\sin x} \cdot \cos x}{\frac{1}{\tan x} \cdot \sec^2 x}$$

$$\lim_{x \to 0^+} \frac{\frac{\cos x}{\sin x}}{\frac{\sec^2 x}{\tan x}} \qquad \cancel{\frac{\cos x}{\sin x}} \cdot \cancel{\tan x}^{\,1} \cdot \frac{1}{\sec^2 x}$$

$$\lim_{x \to 0^+} \frac{1}{\sec^2 x}$$

$$\lim_{x \to 0^+} \cos^2 x = 1$$

6. 9. 10.

6. $\displaystyle\int_0^1 \frac{x+1}{\sqrt{x^2+2x}}\,dx$

$$\lim_{b \to 0} \int_b^1 \frac{x+1}{\sqrt{x^2+2x}}\,dx$$

$u = x^2 + 2x$

$du = 2x+2 \; dx$

$\frac{du}{2x+2} = dx$

$\frac{du}{2(x+1)} = dx$

$$\int \frac{\cancel{x+1}}{\sqrt{u}} \cdot \frac{du}{2(\cancel{x+1})}$$

$$\frac{1}{2} \int u^{-\frac{1}{2}}\,du$$

$$\frac{1}{\cancel{2}}\left(\cancel{2}\, u^{\frac{1}{2}}\right)$$

$$\lim_{b \to 0} \left(\sqrt{x^2+2x} \,\Big|_b^1 \right)$$

$$\lim_{b \to 0} \left(\sqrt{1^2+2(1)} - \cancel{\sqrt{b^2-2b}}^{\,0} \right)$$

$= \sqrt{3}$

9. $\int_0^3 \frac{dx}{x-2}$

$$\lim_{b\to 2}\int_0^b \frac{1}{x-2}\,dx + \lim_{b\to 2}\int_b^3 \frac{1}{x-2}\,dx$$

$$\lim_{b\to 2}\left(\ln|x-2|\Big|_0^b\right) + \lim_{x\to 2}\left(\ln|x-2|\Big|_b^3\right)$$

$$\lim_{b\to 2}\left(\ln|b-2| - \ln|0-2|\right) + \lim_{b\to 2}\left(\ln\cancel{|3-2|}^{0} - \ln|b-2|\right)$$

$$-\infty - \ln 2 +$$

$$\ln\left|\frac{b-2}{0-2}\right|$$

10. $\int_{-1}^{8} \frac{dx}{\sqrt[3]{x}}$

$$\lim_{b\to 0}\int_{-1}^{b} x^{-\frac{1}{3}}\,dx + \lim_{b\to 0}\int_b^8 x^{-\frac{1}{3}}\,dx$$

$$\lim_{b\to 0}\left(\frac{3}{2}x^{\frac{2}{3}}\Big|_{-1}^{b}\right) + \lim_{b\to 0}\left(\frac{3}{2}x^{\frac{2}{3}}\Big|_b^8\right)$$

$$\lim_{b\to 0}\left(\cancel{\frac{3}{2}b^{\frac{2}{3}}} - \frac{3}{2}(-1)^{\frac{2}{3}}\right) + \lim_{b\to 0}\left(\frac{3}{2}(8)^{\frac{2}{3}} - \cancel{\frac{3}{2}b^{\frac{2}{3}}}\right)$$

$$-\frac{3}{2} + 6 = \frac{9}{2}$$

39

$$\int_0^\infty \frac{16 \tan^{-1} v}{1+v^2}\, dv$$

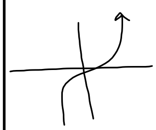
$$16 \lim_{b\to\infty} \int_0^b \frac{\tan^{-1} v}{1+v^2}\, dv \qquad u = \tan^{-1} v \qquad du = \frac{1}{1+v^2}\, dv$$

$$16 \int u\, du$$

$$8u^2$$

$$\lim_{b\to\infty} \left(8 \left(\tan^{-1} v\right)^2 \Big|_0^b \right)$$

$$\lim_{b\to\infty} \left(8 \left(\tan^{-1} b\right)^2 - 8\left(\tan^{-1}(0)\right)^2 \right)$$



$$8\left(\frac{\pi^2}{4}\right) - 0$$

$$2\pi^2$$

$$\frac{1}{x} < \frac{2+\cos x}{x}$$

37.

$$\int_0^\infty \frac{ds}{(1+s)(\sqrt{s})}$$

$u^2 = s$

$u = \sqrt{s}$

$du = \frac{1}{2} s^{-\frac{1}{2}} ds$

$du = \frac{1}{2\sqrt{s}} ds$

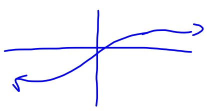
$2\sqrt{s}\, du = ds$

$$\int \frac{2\cancel{\sqrt{s}}\, du}{(1+u^2)\cancel{\sqrt{s}}}$$

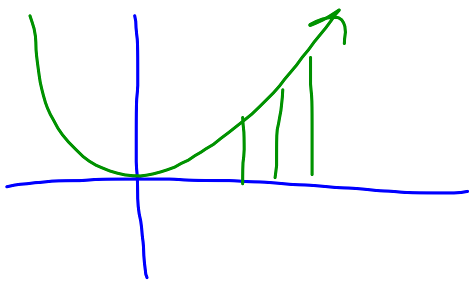
$$2\int \frac{1}{1+u^2}\, du$$

$$\lim_{b\to\infty} \left(2\tan^{-1}(\sqrt{s}) \Big|_0^b \right)$$

$$\lim_{b\to\infty} \left(2\tan^{-1}(\sqrt{b}) - 2\left(\cancel{\tan^{-1}(\sqrt{0})}\right)\right)$$



$2\left(\frac{\pi}{2}\right) = \pi$



$$\lim_{x\to\infty}$$