

3.

$$\lim_{x \rightarrow \infty} \frac{e^x}{e^{\cos x}}$$

$x > \cos x$ as $x \rightarrow \infty$
because $-1 \leq \cos x \leq 1$

7.

$$\lim_{x \rightarrow \infty} \frac{\ln x}{\sqrt[3]{x}} = \frac{\frac{1}{x}}{\frac{1}{3} x^{-\frac{2}{3}}} = \frac{\frac{1}{x}}{\frac{1}{3x^{\frac{2}{3}}}} \cdot \frac{3x^{\frac{2}{3}}}{1}$$

$$\lim_{x \rightarrow \infty} \frac{3}{x^{\frac{1}{3}}} = 0$$

$$\lim_{x \rightarrow \infty} \frac{x+2}{x^3+3x} = 0$$

$$\lim_{x \rightarrow \infty} \frac{\frac{x^4}{x^4} + \frac{5x}{x^4}}{\frac{x^3}{x^4}} = \infty$$

8.3

15. $\lim_{x \rightarrow \infty} \frac{e^x}{\sqrt{1+x^4}} = \infty$

29. e^x x^x $(\ln x)^x$ $e^{\frac{x}{2}}$
 slowest to fastest

$e^{\frac{x}{2}}$ $e^x (\ln x)^x$

$\lim_{x \rightarrow \infty} \frac{(\ln x)^x}{e^x} = \infty$ $\lim_{x \rightarrow \infty} \frac{(\ln x)^x}{x^x}$

$\lim_{x \rightarrow \infty} \left(\frac{\ln x}{e}\right)^x$ $\lim_{x \rightarrow \infty} \left(\frac{\ln x}{x}\right)^x$

33.

33.

$$\begin{array}{ccc}
 \underline{3^x} & \sqrt{9^x + 2^x} & \sqrt{9^x - 4^x} \\
 & (9^x)^{\frac{1}{2}} & \\
 & (9^{\frac{1}{2}})^x & \\
 & 3^x &
 \end{array}$$

11. $\int_{-\infty}^{-2} \frac{2 dx}{x^2 - 1}$

$$\lim_{b \rightarrow -\infty} \int_b^{-2} \frac{2 dx}{x^2 - 1}$$

$$\frac{2}{x^2 - 1} = \frac{A(x-1) + B(x+1)}{x^2 - 1}$$

$$2 = A(x-1) + B(x+1)$$

$$\lim_{b \rightarrow -\infty} \int_b^{-2} \left(\frac{-1}{x+1} + \frac{1}{x-1} \right) dx$$

$$x=1 \quad 2 = 2B$$

$$B=1$$

$$x=-1 \quad 2 = -2A$$

$$A=-1$$

$$\lim_{b \rightarrow -\infty} \left(-\ln|x+1| + \ln|x-1| \right) \Big|_b^{-2}$$

$$\lim_{b \rightarrow -\infty} \left(-\ln|2+1| + \ln|-2-1| - \left(-\ln|b+1| + \ln|b-1| \right) \right)$$

$$\lim_{b \rightarrow -\infty} \left(\ln 3 + \left(\ln \left| \frac{b+1}{b-1} \right| \right) \right) = \ln 3$$

23.

$$\int_{-\infty}^{\infty} \frac{dx}{e^x + e^{-x}}$$

$$\int_{-\infty}^0 \frac{dx}{e^x + e^{-x}} + \int_0^{\infty} \frac{dx}{e^x + e^{-x}}$$

$$\lim_{b \rightarrow -\infty} \int_b^0 \frac{dx}{e^x + e^{-x}} + \lim_{b \rightarrow \infty} \int_0^b \frac{dx}{e^x + e^{-x}}$$

$$\frac{1}{e^x \frac{e^x}{e^x} + \frac{1}{e^x}}$$

$$\frac{1}{e^{2x} + 1}$$

$$\int \frac{e^x}{e^{2x} + 1} \quad u = e^x$$

$$\frac{du}{e^x} = e^x dx \quad du = u du$$

$$\int \frac{1}{u+1} \cdot \frac{du}{u}$$

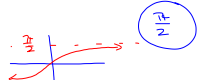
$$\int \frac{du}{u^2+1}$$

$$\tan^{-1} u$$

$$\lim_{b \rightarrow -\infty} (\tan^{-1} e^x) \Big|_b^0 + \lim_{b \rightarrow \infty} (\tan^{-1} e^x) \Big|_0^b$$

$$\lim_{b \rightarrow -\infty} (\tan^{-1} e^0 - \tan^{-1} e^b) + \lim_{b \rightarrow \infty} (\tan^{-1} e^b - \tan^{-1} e^0)$$

$$\frac{1}{e^{\infty}}$$

$$\left(\frac{\pi}{4} - 0\right) + \left(\frac{\pi}{2} - \frac{\pi}{4}\right)$$


27.

$$\int_0^1 \frac{x+1}{\sqrt{x^2+2x}} dx$$

a. improper because VA @ x=0.

$$x(x+2)$$

$$\lim_{b \rightarrow 0} \int_b^1 \frac{x+1}{\sqrt{x^2+2x}} dx$$

$$u = x^2 + 2x$$

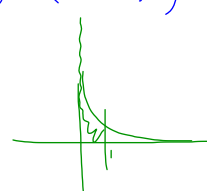
$$du = 2x + 2 dx$$

$$\frac{du}{2(x+1)} dx$$

$$\int \frac{\cancel{x+1}}{u^{\frac{1}{2}} \cdot 2(\cancel{x+1})} du$$

$$\frac{1}{2} \int u^{-\frac{1}{2}} du = \frac{1}{2} (2u^{\frac{1}{2}})$$

$$\lim_{b \rightarrow 0} \left((x^2+2x)^{\frac{1}{2}} \Big|_b^1 \right)$$

$$\lim_{b \rightarrow 0} \left((3)^{\frac{1}{2}} - (b^2+2b)^{\frac{1}{2}} \right) = \sqrt{3}$$


17.

$$\lim_{x \rightarrow \infty} \frac{x \ln x - x}{e^x} = \frac{\cancel{x} \frac{1}{x} + \ln x - 1}{e^x}$$

$$\frac{\cancel{e^x} \rightarrow \infty}{\frac{1}{\cancel{x}} \rightarrow 0}$$