43.

$$a_n = \frac{1}{n!}$$

$$0 < \frac{1}{n!} \leq \frac{1}{n}$$

$$a_n \to 0$$

$$\frac{1}{n!} \to 0$$

19

$$(2, -2)$$
$$(5, 7)$$

8.2 L'Hopital's Rule

Indeterminate forms: $\frac{0}{0}$ or $\frac{\infty}{\infty}$

$$\lim_{x\to a}\frac{f(x)}{g(x)}=\lim_{x\to a}\frac{f'(x)}{g'(x)}$$

$$\lim_{x\to 0}\frac{\sin x}{x} \quad \frac{0}{0}\checkmark = \lim_{x\to 0}\frac{\cos x}{1} = 1$$

Graph $\frac{\sin x}{x}$ and $\frac{\cos x}{1}$

How does this support L'Hopital's rule?

$$\lim_{x\to 1}\frac{\ln x}{x-1} \quad \frac{0}{0}\checkmark = \lim_{x\to 1}\frac{\frac{1}{x}}{1} = 1$$

$$\lim_{x\to 0}\frac{\sqrt{1+x}-1}{x} \quad \frac{0}{0}\checkmark = \lim_{x\to 0}\frac{\frac{1}{2\sqrt{1+x}}}{1} = \frac{1}{2}$$

$$\lim_{x\to \pi}\frac{\pi-x}{\sin x} \quad \frac{0}{0}\checkmark \quad \lim_{x\to\pi}\frac{-1}{\cos x} = \frac{-1}{-1} = 1$$

$$\lim_{x\to 0}\frac{4\cancel{x}}{x^{\cancel{2}}} \quad \frac{0}{0}\checkmark \quad \lim_{x\to 0}\frac{4}{2x} = \text{DNE}$$

$$\lim_{x\to 0}\frac{4}{x}$$

$$\lim_{x\to \frac{\pi}{2}^-}\frac{\sec x}{1+\tan x} \quad \frac{\infty}{\infty}\checkmark = \lim_{x\to\frac{\pi}{2}^-}\frac{\cancel{\sec x}\tan x}{\sec^{\cancel{2}} x}$$

$$\lim_{x\to\frac{\pi}{2}^-}\frac{\tan x}{\sec x} \quad \frac{\frac{\sin x}{\cancel{\cos x}}}{\frac{1}{\cancel{\cos x}}}$$

$$\lim_{x\to\frac{\pi}{2}^-}\sin x = 1$$

Other indeterminate forms

$\infty - \infty \quad \infty \cdot 0$ change to a quotient

$$\lim_{x\to\infty} x\sin\frac{1}{x} \qquad \lim_{x\to\infty} \frac{\sin\frac{1}{x}}{\frac{1}{x}} \qquad \frac{0}{0}\checkmark$$

$$\lim_{x\to\infty} \frac{\left(\cos\frac{1}{x}\right)\cancel{\left(-\frac{1}{x^2}\right)}}{\cancel{-\frac{1}{x^2}}} = \lim_{x\to\infty}\left(\cos\underset{0}{\frac{1}{x}}\right) = 1$$

$$\lim_{x\to\frac{\pi}{2}^-}\left(\sec x - \tan x\right)$$

$$\frac{1}{\cos x} - \frac{\sin x}{\cos x}$$

$$\lim_{x\to\frac{\pi}{2}^-} \frac{1-\sin x}{\cos x} \qquad \frac{0}{0}\checkmark$$

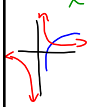
$$\lim_{x\to\frac{\pi}{2}^-} \frac{-\cos x}{-\sin x} = \frac{0}{-1} = 0$$

Other indeterminate forms

$1^\infty \quad 0^0 \quad \infty^0$ use logs/exponential rules

$$\lim_{x\to 0^+} \ln x^x = \ln y$$

$$\lim_{x\to 0^+} x\ln x = \ln y$$



$$\lim_{x\to 0^+} \frac{\ln x}{\frac{1}{x}} \quad \frac{\infty}{\infty}\checkmark \quad \ln y$$

$$\lim_{x\to 0^+} \frac{\frac{1}{x}}{\cancel{-\frac{1}{x^2}}}\cdot\frac{-x^2}{1} = \ln y$$

$$\lim_{x\to 0^+} -x = \ln y$$

$$0 = \ln y$$

$$1 = y$$

$$\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{x} = y$$

$$\lim_{x\to\infty} \frac{\ln\left(1+\frac{1}{x}\right)}{\frac{1}{x}} \overset{\checkmark \; \frac{0}{0}}{=} \ln y$$

$$\lim_{x\to\infty} \frac{\frac{1}{\left(1+\frac{1}{x}\right)}\cdot \cancel{-\frac{1}{x^2}}}{\cancel{-\frac{1}{x^2}}} = \ln y$$

$$\lim_{x\to\infty} \frac{1}{1+\cancel{\frac{1}{x}}^{\to 0}} = \ln y$$

$$1 = \ln y$$

$$e = y$$

$$\lim_{x\to\infty}\left(1-\frac{3}{x}\right)^{2x}$$

$$\lim_{x\to\infty} \frac{\ln\left(1-\frac{3}{x}\right)}{\frac{1}{2x}} \overset{\checkmark \; \frac{0}{0}}{=} \ln y$$

$$\lim_{x\to\infty} \frac{\left(\frac{1}{1-\frac{3}{x}}\right)\frac{3}{\cancel{x^2}}\cdot -\frac{4\cancel{x^2}}{2}}{\cancel{\frac{-1}{(2x)^2}\cdot 2}} = \ln y$$

$$\lim_{x\to\infty} \frac{-6}{1-\cancel{\frac{3}{x}}^{\to 0}} = \ln y$$

$$-6 = \ln y$$

$$y = e^{-6}$$