8.1 Sequences

Sequence Vocab.

sequence -

finite

infinite

Geometric Sequence

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geometric - sequence with a common ratio (quotient) between successive terms (repeated multiplication)
exponential

explicit rule: $a_n = a_1 \bullet r^{(n-1)}$

r = common ratio
n = term number
a = term

recursive rule: $\underset{n+1}{a_n} = \underset{n}{a_{n-1}} \bullet r \qquad n \geq 2$

$a_1 = \#$

Find the common ratio, a recursive rule, and an explicit rule for the following sequences:

$4, -2, 1, -\frac{1}{2}, \ldots$

$r = -\frac{1}{2}$

rec. $\begin{cases} a_1 = 4 \\ a_n = a_{n-1} \cdot -\frac{1}{2} \end{cases}$

$a_1 = 4$
$a_2 = 4 \cdot -\frac{1}{2} = -2$
$a_3 = -2 \cdot -\frac{1}{2} = 1$

$a_n = 4\left(-\frac{1}{2}\right)^{n-1}$

$$\frac{4\left(-\frac{1}{2}\right)^{n}}{\left(-\frac{1}{2}\right)^{1}} = -8\left(-\frac{1}{2}\right)^{n}$$

Graph it:

The second and fifth terms of a geometric sequence are 6 and -48, respectively. Find an explicit expression for the nth term.

$6 = a_1 (r)^{2-1}$

$-48 = a_1 (r)^{5-1}$

$\frac{6}{r} = a_1 \cdot \frac{\cancel{r}}{\cancel{r}}$

$-48 = a_1 \cdot r^4$

$-48 = \frac{6}{r} \cdot r^4$

$-48 = 6r^3$

$-8 = r^3$

$-2 = r$

$\frac{6}{-2} = a_1 = -3$

$a_n = -3(-2)^{n-1}$

Convergence/Divergence
of an infinite sequence

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if $\{a_n\}$ is a sequence - consider $\lim_{n\to\infty} a_n$

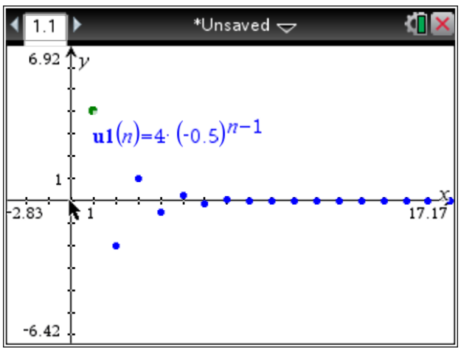
convergence: if the limit is a finite number - the sequence converges

$-1 < r < 1$

divergence: if the limit is infinite or non-existent - the sequence diverges

$r > 1$

$r < -1$



Determine whether the sequence converges or diverges. If it converges, give the limit.

2, 4, 6, 8, 10, ... Divergent

$\frac{1}{1}, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots\ldots, \frac{1}{n}, \ldots\ldots$ Convergent $\lim_{n \to \infty} f(n) = 0$

$-1, 1, -1, 1, -1, \ldots\ldots$ Divergent

Determine if the following sequence converges or diverges. If it converges, then find its limit. Use graphical or symbolic methods.

$a_n = \frac{-5n+7}{-7n}$ Convergent: $\frac{5}{7}$

$a_n = \frac{1+2n}{2n^2+1}$ Convergent: 0

$$\frac{5x^2+2}{x+1}$$

Show the sequence converges and find its limit.

$$a_n = \frac{\sin^3 n}{n^3}$$

$$\frac{-1}{n^3} \le \frac{\sin^3 n}{n^3} \le \frac{1}{n^3}$$

$$\frac{-1}{n^3} \to 0 \qquad \frac{1}{n^3} \to 0$$

$$\therefore \frac{\sin^3 n}{n^3} \to 0$$

$$0 \leq \frac{1}{2^n} \leq \frac{1}{n}$$

$0 \to 0$ $\frac{1}{n} \to 0$

$\therefore$ converges & $\frac{1}{2^n} \to 0$