

1. Let R be the shaded region in the first quadrant enclosed by the graphs of $y = e^{-x^2}$, $y = 1 - \cos x$, and the y -axis, as shown in the figure above.

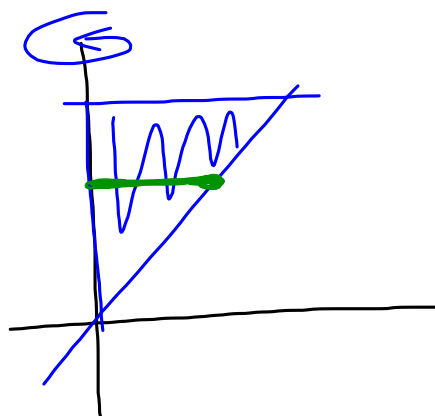
(a) Find the area of the region R .

$$\int_0^{0.942} e^{-x^2} - (1 - \cos x) dx$$

VOLUME: DISKS

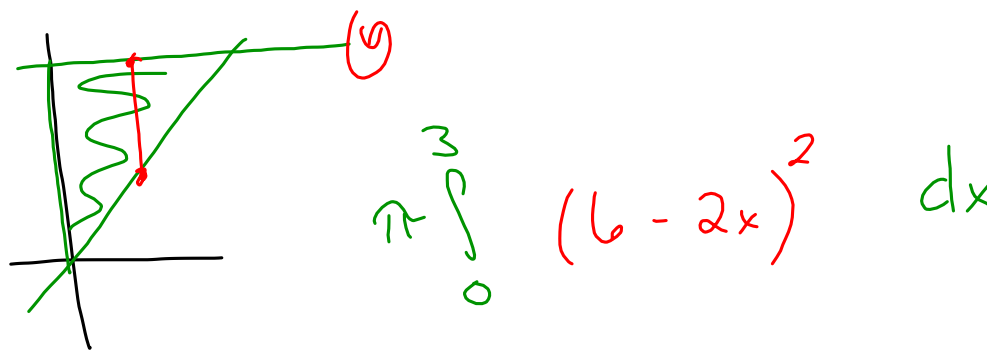
Ex 1: Find the volume if the region enclosing $y = 2x$, $y = 6$, $x = 0$ is rotated about the y axis

2



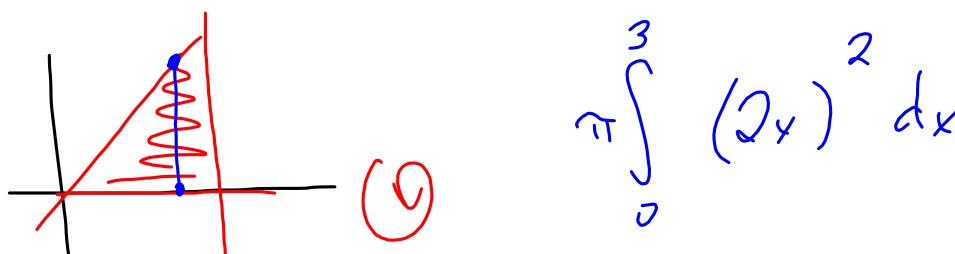
$$\pi \int_0^6 \left(\frac{y}{2}\right)^2 dy$$

Ex 2: Find the volume if the region enclosing $y = 2x, y = 6, x = 0$ is rotated about $y=6$

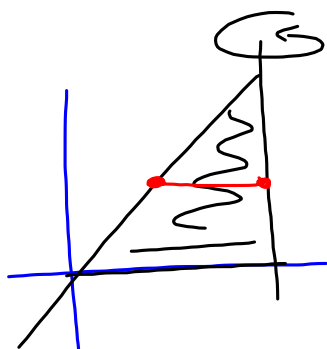


VOLUME: DISKS

Ex 1: Find the volume if the region enclosing $y = 2x, y = 0, x = 3$ is rotated about the x axis



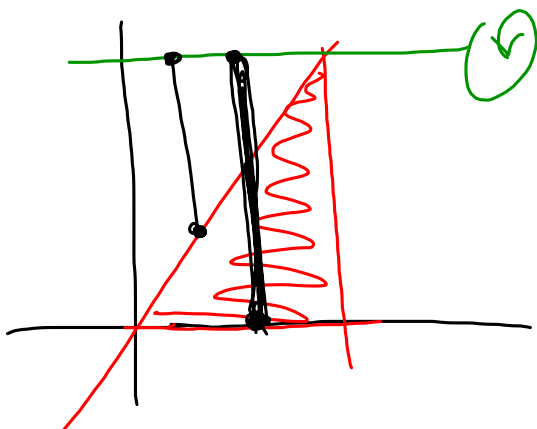
Ex 2: Find the volume if the region enclosing $y = 2x, y = 0, x = 3$ is rotated about $x = 3$



$$\pi \int_0^6 \left(3 - \frac{y}{2} \right)^2 dy$$

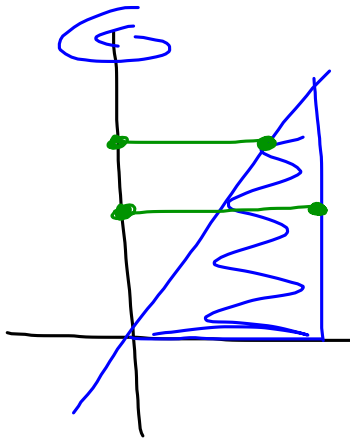
VOLUME: WASHERS

Ex 1: Find the volume if the region enclosing $y = 2x, y = 0, x = 3$ is rotated about $y = 6$



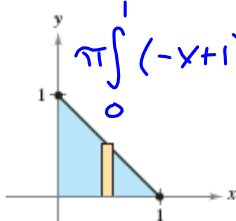
$$\pi \int_0^3 (6)^2 - (6 - 2x)^2 dx$$

Ex 1: Find the volume if the region enclosing $y = 2x, y = 0, x = 3$ is rotated about the y -axis



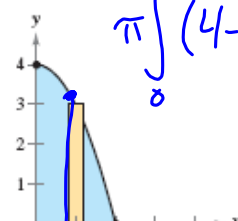
$$\pi \int_0^6 (3)^2 - \left(\frac{y}{2}\right)^2 dy$$

1. $y = -x + 1$



$$\pi \int_0^1 (-x+1)^2 dx$$

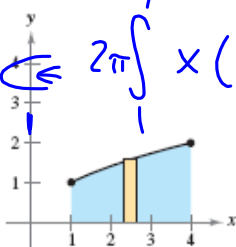
2. $y = 4 - x^2$



$$\pi \int_0^2 (4-x^2-2)^2 - (2)^2 dx$$

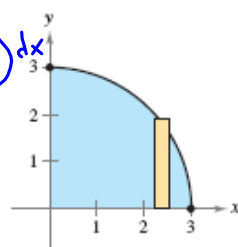
Shell

3. $y = \sqrt{x}$



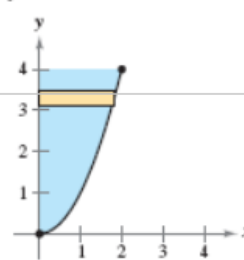
$$2\pi \int_1^4 x(\sqrt{x}) dx$$

4. $y = \sqrt{9-x^2}$

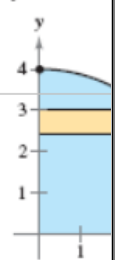


In Exercises 7-10, set up and evaluate the integral volume of the solid formed by revolving the y -axis.

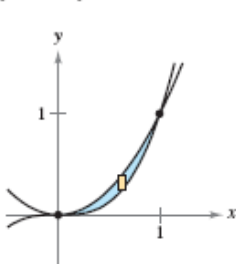
7. $y = x^2$



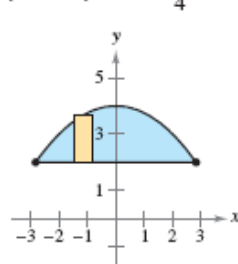
8. $y = \sqrt{16-x^2}$



5. $y = x^2, y = x^3$



6. $y = 2, y = 4 - \frac{x^2}{4}$



9. $y = x^{2/3}$

$2\pi \int_0^1 y (y^{3/2}) dy$

10. $x = -y^2 + 4y$

In Exercises 11–14, find the volume of the solid generated by revolving the region bounded by the graphs of the equations about the given lines.

11. $y = \sqrt{x}$, $y = 0$, $x = 4$

(a) the x -axis (b) the y -axis
 (c) the line $x = 4$ (d) the line $x = 6$

Work: Find the work required to pull a leaky 5 lb pail with 6 ft³ of water at the start. It loses $\frac{1}{3}$ of the water over 10 ft.

Pail:

$W = 5 \cdot 10 =$
50 ft.-lb.

Water:

$W = \int_0^{10} 6 \cdot 4 \left(6 - \frac{2}{10}x\right)$

Work

Move a spring 3 inches rF

300 lb moves a spring 1 inch.

$$F = kx$$

$$300 = k \cdot 1 \quad k = 300$$

$$F = 300x$$

$$\int_0^3 300x \, dx$$

Find the work done in pulling
a leaky pail that weighs 5 lbs.

It begins w/ 3 gallons of water
and holds $\frac{1}{2}$ that amount after 10 ft.

8 lb./gal.

Pail

Water

50 ft. lb.

$$+ \int_0^{10} 8 \left(3 - \frac{1.5}{10} x \right) dx$$

Work:

30 ft. Rope that weight

.2 lb Find the work to
ft.

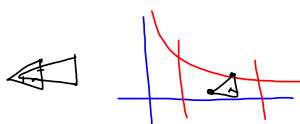
lift 20 ft. of rope.

$$W = \int_0^{20} .2 \underbrace{(30 - x)}_{\text{length}} dx$$

$$w = .2x$$

$\frac{1}{x}$ from 1 to 3 has cross-
sections \perp to x -axis in the
shape of isosceles rt. Δ 's.
(w/ base \perp)

find Volume:



$$\int \frac{1}{2} s^2$$

$$\int_1^3 \frac{1}{2} \left(\frac{1}{x}\right)^2 dx$$

$$\frac{1}{2} \int_1^3 x^{-2} dx$$

$$\frac{1}{2} \left(-x^{-1} \right) \Big|_1^3$$

Attachments

Calc - WS on Differential Equations.doc