9.

$y = \sec x$

$$\int_{-\pi/3}^{\pi/3} \sqrt{1 + (\sec x \tan x)^2}\, dx$$

17.

$$x = \int_0^y \sqrt{\sec^4 t - 1}\, dt \qquad -\frac{\pi}{4} \le y \le \frac{\pi}{4}$$

$$\frac{dx}{dy} = \sqrt{\sec^4 y - 1}$$

$$\int_{-\pi/4}^{\pi/4} \sqrt{1 + \left(\sqrt{\sec^4 y - 1}\right)^2}\, dy$$

$$\int_{-\pi/4}^{\pi/4} \sqrt{\sec^4 y}\, dy$$

$$\int_{-\pi/4}^{\pi/4} \sec^2 y\, dy$$

$$\tan y \Big|_{-\pi/4}^{\pi/4} = 2$$

19. thru (1,1)

$$L = \int_1^4 \sqrt{1 + \frac{1}{4x}}\, dx$$

$$\sqrt{\left(\frac{dy}{dx}\right)^2} = \sqrt{\frac{1}{4x}}$$

$$\int dy = \int \frac{1}{2\sqrt{x}}\, dx$$

$$y = \frac{1}{2}\int x^{-\frac{1}{2}}\, dx$$

$$y = \frac{1}{2}\left(2x^{\frac{1}{2}}\right) + C$$

$$y = x^{\frac{1}{2}} + C$$

$$1 = 1 + C$$

$$C = 0$$

$$y = x^{\frac{1}{2}}$$

7.5 Applications from Science and Stats

Work done by a constant force: $W = F \bullet d$

Work done by a variable force: $W = \int_a^b F(x)dx$

A leaky bucket weighs 22N empty. It is lifted from the ground at a constant rate to a point 20m above the ground by a rope weighing 0.4 N/m. The bucket starts with 70N of water but it leaks at a constant rate and just finishes draining as the bucket reaches the top. Find the amount of work done.

W_{bucket} constant

$22 \cdot 20$

$W = 440$ Nm

W_{water} variable (leaking)

$$\int_0^{20} \left(70 - \frac{7}{2}x\right) dx$$

$$70x - \frac{7x^2}{4}\Big|_0^{20}$$

$1400 - 700$

700 Nm

W_{rope} variable (last rope assume pulled)

$$\int_0^{20} .4(20 - x)\,dx$$

$$.4\int_0^{20} 20 - x$$

$$.4\left(20x - \frac{x^2}{2}\right)_0^{20}$$

$.4(400 - 200)$

$.4(200)$

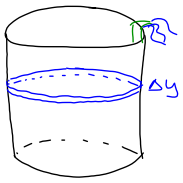
80 Nm

Total W = 440 + 700 + 80

1220 Nm

How much work does it take to pump all the water over the rim of a cylindrical tank of height 10ft and diameter 10ft?

density water = $62.4 \frac{lb}{ft^3}$



$W_{slab} = d \cdot \pi r^2 \Delta y$

$W_{slab} = 62.4\pi(5)^2 \Delta y$

$$Work = \int_a^b \text{weight} \cdot \text{height w/in cylinder}$$

$$W = \int_0^{10} \underbrace{y}_{\text{height in cyl.}} \underbrace{62.4\pi(25)\,dy}_{\text{weight}}$$

$$W = (62.4)(25)(\pi)\int_0^{10} y\,dy$$

$$\left(\frac{y^2}{2}\right)_0^{10}$$

$W = (62.4)(25)(50)\pi$

$= 245044.227$ ft. lb.