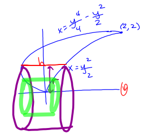
34a.



$y=0$

$$\int 2\pi r h \, dy$$

$$2\pi\int_0^2 (y)\left(\frac{y^2}{2} - \left(\frac{y^4}{4} - \frac{y^2}{2}\right)\right) dy$$

$$2\pi\int_0^2 y\left(y^2 - \frac{y^4}{4}\right) dy$$

$y \cdot \frac{y^2}{2}$

$$2\pi\int_0^2 \left(y^3 - \frac{y^5}{4}\right) dy$$

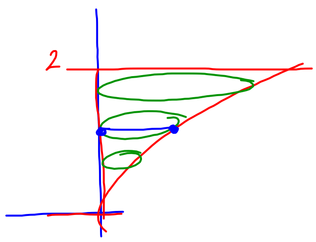
$$2\pi\left(\frac{y^4}{4} - \frac{y^6}{24}\right)\Big|_0^2$$

$$\pi\left(\frac{y^4}{2} - \frac{y^6}{12}\right)\Big|_0^2$$

$$\pi\left(8 - \frac{16}{3}\right) = \frac{8}{3}\pi$$

41.

$x = \sqrt{5}\, y^2$



$A_{\text{cross-section}} = \pi r^2$

$$\int_0^2 \pi \left(\frac{\sqrt{5}\, y^2}{2}\right)^2 dy$$

$$\int_0^2 \pi \frac{5y^4}{4} \, dy$$

$$\frac{5\pi}{4}\int_0^2 y^4 \, dy$$

$$\frac{\pi}{4}\left(\frac{y^5}{\ }\right)\Big|_0^2$$

7.4 Length of a Curve

$$d = \sqrt{\Delta x^2 + \Delta y^2}$$

$$\lim_{n\to\infty} \sum_{i=1}^{n} \sqrt{\Delta x_i^2 + \Delta y_i^2}$$

$$\text{Curve Length} = \int_a^b \sqrt{\frac{\Delta x^2}{\Delta x^2} + \frac{\Delta y^2}{\Delta x^2}}\, dx$$

$$C.L. = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

Definition of Arc Length

$$L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$L = \int_c^d \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$

$$x = y^2$$

$$\frac{dx}{dy} = 2y$$

Find the exact length of the curve $y = x^2$ $\quad 0 \le x \le 4$

$$\frac{dy}{dx} = 2x$$

$$C.L. = \int_0^4 \sqrt{1 + (2x)^2}\, dx$$

$$= \int_0^4 (1 + 4x^2)^{\frac{1}{2}}\, dx$$

$$= 16.819$$

Find the exact length of the curve $y = \frac{4\sqrt{2}}{3} x^{\frac{3}{2}} - 1$ $\quad 0 \le x \le 1$

$$\frac{dy}{dx} = \frac{\overset{2}{\cancel{4}}\sqrt{2}}{\cancel{3}}\left(\frac{\cancel{3}}{\cancel{2}} x^{\frac{1}{2}}\right)$$

$$\frac{dy}{dx} = 2\sqrt{2}\, x^{\frac{1}{2}}$$

$$L = \int_0^1 \sqrt{1 + (2\sqrt{2}x^{\frac{1}{2}})^2}\, dx$$

$$\int_0^1 (1 + 8x)^{\frac{1}{2}}\, dx$$

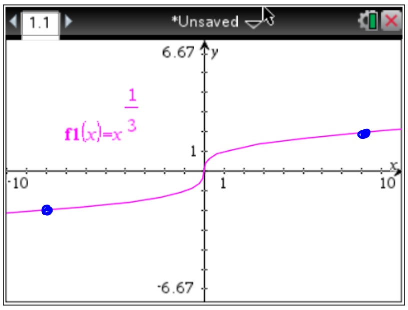
$$\left.\frac{\cancel{2}\,(1+8x)^{\frac{3}{2}}}{3\ \ \underset{4}{\cancel{8}}}\right|_0^1$$

$$\left.\frac{1}{12}(1+8x)^{\frac{3}{2}}\right|_0^1$$

$$\frac{1}{12}(27 - 1) = \frac{26}{12} = \frac{13}{6}$$

A vertical tangent

Find the length of the curve $y = \sqrt[3]{x}$ between (-8, -2) and (8, 2)



$x = y^3$

$\frac{dx}{dy} = 3y^2$

$\int_{-2}^{2} \sqrt{1 + (3y^2)^2}\, dy$

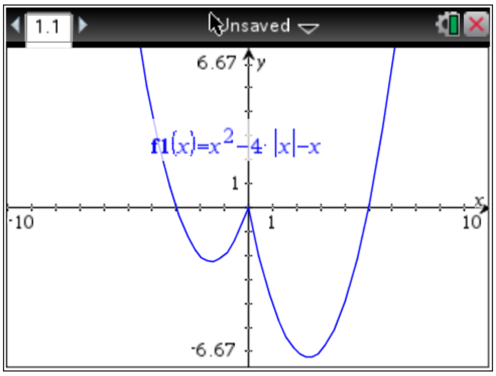
$\int_{-2}^{2} \sqrt{1 + 9y^4}\, dy$

$= 17.261$

A cusp

Find the length of the curve $y = x^2 - 4|x| - x$

from x = - 4 to x = 4



$\int_{-4}^{0} + \int_{0}^{4}$

$$y^2 + 2y = 2x + 1$$ (-1, -1) to (7, 3)

$$y^2 + 2y - 1 = 2x$$

$$x = \frac{y^2}{2} + \frac{2y}{2} - \frac{1}{2}$$

$$\frac{dx}{dy} = y + 1$$

$$\int_{-1}^{3} \sqrt{1 + (y+1)^2}\, dy$$

$$= 9.294$$