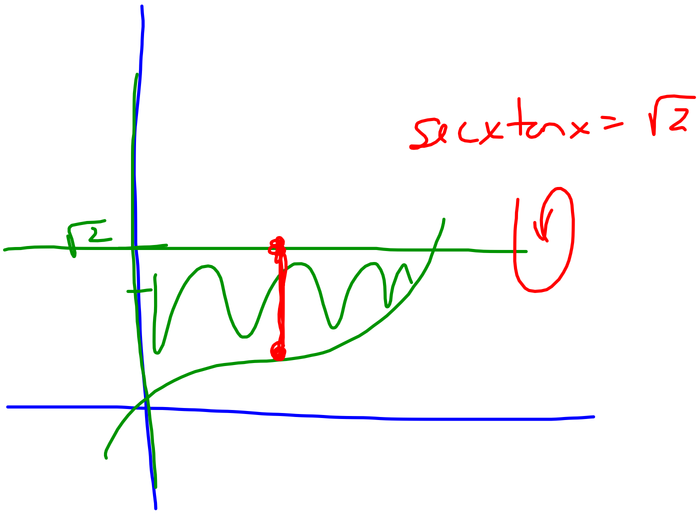
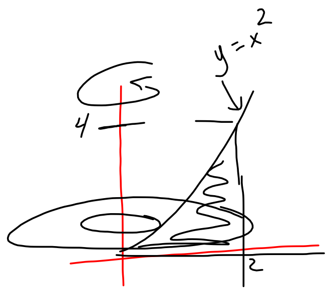
21.




$$\pi \int_0^{.785} \left(\sqrt{2} - \sec x \tan x\right)^2 dx$$

27.




$$\pi \int_0^4 (2)^2 - (\sqrt{y})^2 dy$$

$$\pi \int_0^4 (4 - y) dy$$

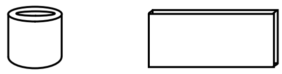
$$\pi \left(4y - \frac{y^2}{2}\right|_0^4$$

$$\pi(16 - 8) - 0$$

$$8\pi$$

7.3c Shells

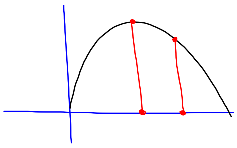
cylindrical shells - shells are like cylindrical tree rings



$\int_a^b 2\pi r h\, dx$

Revolve the region bounded by $y = 3x - x^2$ and the x--axis about the y--axis

$x(3-x)$



$2\pi \int_0^3 (x)(3x - x^2)\, dx$

$2\pi \int_0^3 3x^2 - x^3 \, dx$

$2\pi \left( x^3 - \frac{x^4}{4} \right)\Big|_0^3$

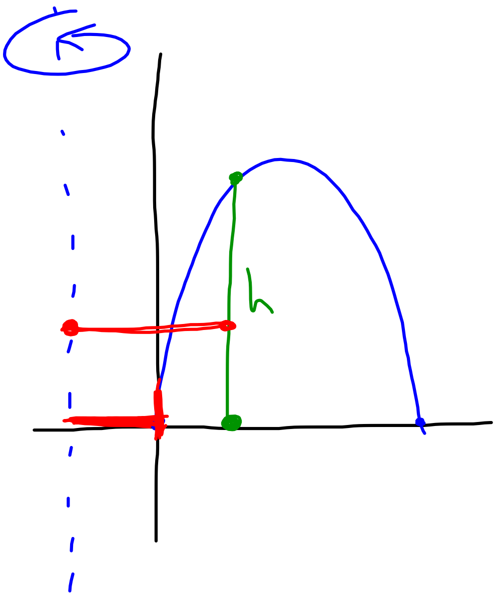
$2\pi \left( 27 - \frac{81}{4} \right) =$

$\pi \left( 54 - \frac{81}{2} \right) = \frac{27\pi}{2}$

notice the axis of rotation is perpendicular to the bounds of integration

Revolve the region bounded by $y = 3x - x^2$ and the x--axis about the line x = -1

$\int 2\pi r h \, dx$





$2\pi \int_0^3 (x+1)(3x - x^2)\, dx$

Find the volume created by rotating the region bounded by

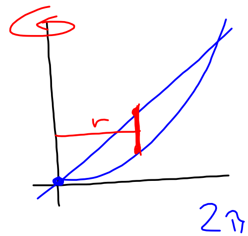
$y = x$ & $y = x^2$ a) about the y-axis

b) about the line x = -2

$x = x^2$

$x^2 - x = 0$

$x(x-1) = 0$




$2\pi r h$

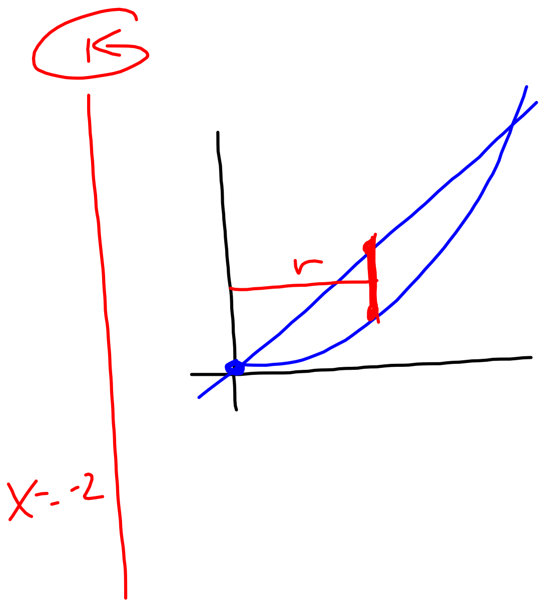
$2\pi \int_0^1 x \, (x - x^2)\, dx$

$2\pi \int_0^1 (x^2 - x^3)\, dx$

$2\pi \left( \frac{x^3}{3} - \frac{x^4}{4} \right) \Big|_0^1$

$2\pi \left( \frac{1}{3} - \frac{1}{4} \right)$

$\pi \left( \frac{2}{3}\cdot 2 - \frac{1}{2}\cdot 3 \right) = \frac{\pi}{6}$

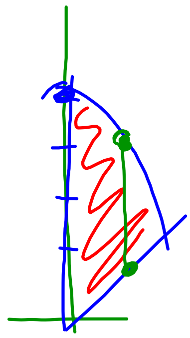



$2\pi \int_0^1 x\,(x - x^2)\, dx$

$2\pi \int_0^1 (x+2)(x - x^2)\, dx$

The region bounded by the curves $y=4-x^2$, $y=x$ and $x=0$ is revolved about the y-axis to form a solid. Use shells to find the volume of the solid.

$2\pi r h$



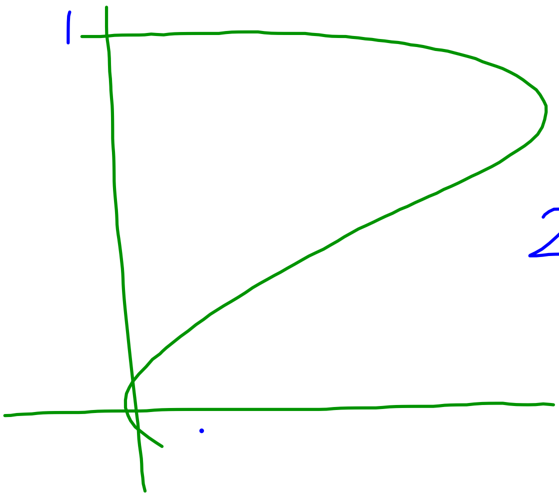
$$2\pi\int_0^{1.56155} x\left((4-x^2)-(x)\right)dx$$

$x=4-x^2$

$x^2+x-4=0$

33a.

$x=12(y^2-y^3)$




$\int 2\pi r h \, dy$

$$2\pi\int_0^1 y\left(12(y^2-y^3)\right)dy$$

1