7.1 Integral as Net Change

The definite integral of a rate of change gives the net change.

displacement:

position:

total distance:

The velocity of a particle moving along the x-axis is given by:

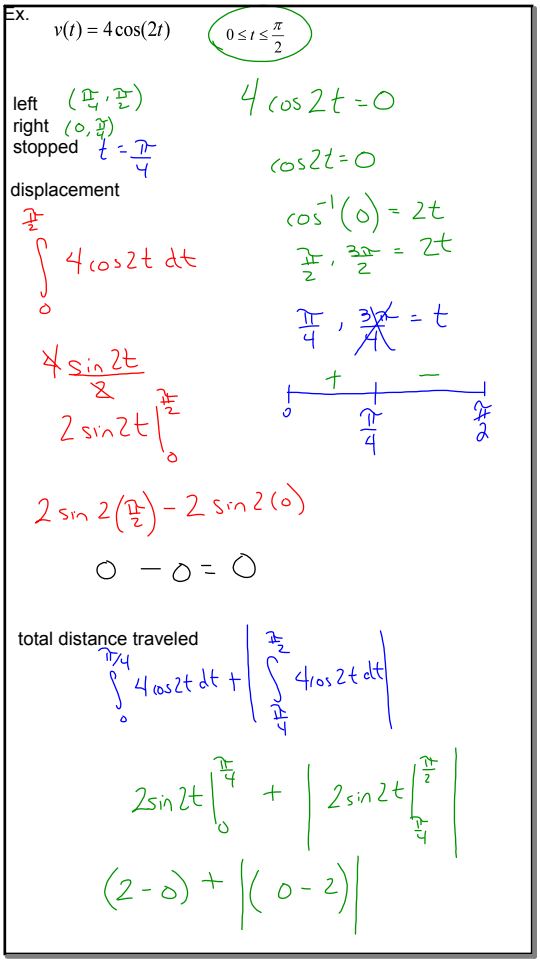
$$v(t) = t^2 - \frac{8}{(t+1)^2} \qquad 0 \le t \le 8$$

a. describe the motion of the particle:

left
right
stopped

b. Initial position $s(0) = 12$
what is the particle's position at t = 1? t = 3?

c. Find the total distance traveled from t = 0 to t = 3.



Integral of a rate of change gives the total accumulation.

Potato Consumption -- From 1970 to 1980 the rate of potato consumption was $C(t) = 2.2 + 1.1^t$ millions of bushels per year, with t being years since the beginning of 1970. How many bushels were consumed from the beginning of 1972 to the end of 1973?

$$\int_2^4 2.2 + 1.1^t\, dt$$

$$\left(2.2t + \frac{1.1^t}{\ln(1.1)}\right)\Big|_2^4$$

Potato Consumption: $C(t) = 5e^{\frac{t}{10}}$

in billions of bushels per year, t = years beginning in 1990

Find the potato consumption from the beginning of 1990 to the end of 1994.

$$\int_0^5 5e^{\frac{t}{10}}\,dt =$$

$$\frac{5e^{\frac{t}{10}}}{\frac{1}{10}}\Bigg|_0^5 = 50e^{\frac{t}{10}}\Big|_0^5$$

$$50e^{\frac{1}{2}} - 50$$

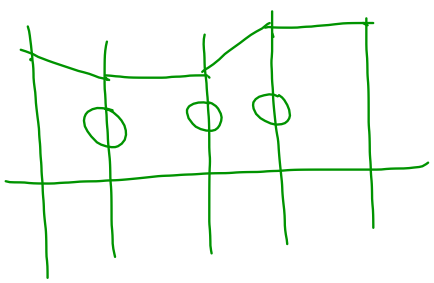
$$32.436\left(\frac{\text{bushels}}{\cancel{\text{yr}}} \cdot \cancel{\text{yr}}\right)$$

A pump connected to a generator operates at a varying rate shown in the table. How many gallons were pumped during the hour?

Time (min)	Rate (gal/min)
0	58
5	60
10	65
15	64
20	58
25	57
30	55
35	55
40	59
45	60
50	60
55	63
60	63

$$5\left(\frac{1}{2}\right)\left(58 + 63 + 2(60 + 65 + \cdots + 60) + 63\right)$$

$$= 3582.5 \text{ gal}$$



Work done by a constant force: $W = F \cdot d$ (displacement)

Work done by a variable force: $W = \int_a^b F(x)dx$

Hooke's Law: $F(x) = kx$ k: spring constant

It takes a force of 9 N to stretch a spring 3 cm. How much force does it take to stretch the spring to 5 cm? How much work is done in stretching the spring to 5 cm?

$9 = k \cdot 3$

$k = 3$

$F(x) = 3x$ $F(5) = 3 \cdot 5 = 15$

$W = \int_0^5 3x\,dx = \frac{3x^2}{2}\Big|_0^5 = \frac{75}{2}$