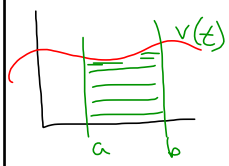
7.1 Integral as Net Change

The definite integral of a rate of change gives the net change.

displacement: final − initial



$\text{displacement} = \int_a^b v(t)\,dt$

position:

$y = y_0 + \int_{x_0}^{x} v(t)\,dt$

total distance:

$\int_a^b |v(t)|\,dt$

$\int_a^c v(t)\,dt + \left| \int_c^b v(t)\,dt \right|$ Left/backward/down

The velocity of a particle moving along the x-axis is given by:

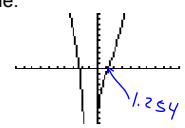
$$v(t) = t^2 - \frac{8}{(t+1)^2} \qquad 0 \le t \le 8$$

a. describe the motion of the particle:

left $[0, 1.254)$

right $(1.254, 8]$

stopped $t = 1.254$



b. Initial position $s(0) = 12$

what is the particle's position at t = 1? t = 3?

$\text{position} = 12 + \int_0^1 t^2 - \frac{8}{(t+1)^2}\,dt = 8.\overline{3}$

$s(3) = 12 + \int_0^3 v(t)\,dt = 15$

c. Find the total distance traveled from t = 0 to t = 3.

w/ calc: $\int_0^3 |v(t)|\,dt$

by hand:

$\left| \int_0^{1.254} v(t)\,dt \right| + \int_{1.254}^3 v(t)\,dt$

Ex. $v(t) = 4\cos(2t)$ $0 \le t \le \frac{\pi}{2}$

left
right
stopped

displacement

total distance traveled

Integral of a rate of change gives the total accumulation.

Potato Consumption -- From 1970 to 1980 the rate of potato consumption was $C(t) = 2.2 + 1.1^t$ millions of bushels per year, with t being years since the beginning of 1970. How many bushels were consumed from the beginning of 1972 to the end of 1973?

Potato Consumption: $C(t) = 5e^{\frac{t}{10}}$

in billions of bushels per year, t = years beginning in 1990

Find the potato consumption from the beginning of 1990 to the end of 1994.

A pump connected to a generator operates at a varying rate shown in the table. How many gallons were pumped during the hour?

Time (min)	Rate (gal/min)
0	58
5	60
10	65
15	64
20	58
25	57
30	55
35	55
40	59
45	60
50	60
55	63
60	63

Work done by a constant force: $W = F \bullet d$

Work done by a variable force: $W = \int_a^b F(x)dx$

Hooke's Law: $F(x) = kx$

It takes a force of 9 N to stretch a spring 3 cm. How much force does it take to stretch the spring to 5 cm? How much work is done in stretching the spring to 5 cm?