1. $\int e^{x^2} x\,dx$

$u = x^2$

$du = 2x\,dx$

$$\int e^u \frac{\cancel{x}\,du}{2\cancel{x}} = \frac{1}{2}\int e^u\,du$$

$$\frac{1}{2}e^{x^2} + C$$

2. $\int 3^x\,dx$

3^x

$3^x \ln 3 \cdot 1$

$$\frac{3^x}{\ln 3 \cdot 1} + C$$

---

3. $\int \csc^2(\sin x)\,\underline{\cos x\,dx}$

$u = \sin x$

$du = \underline{\cos x\,dx}$

$$\int \csc^2 u\,du$$

$$-\cot u + C$$

$$-\cot(\sin x) + C$$

4. $\int x^3 \cos 8x \, dx$

x^3	$+$	$\cos 8x$
$3x^2$	$-$	$\frac{1}{8}\sin 8x$
$6x$	$+$	$-\frac{1}{8^2}\cos 8x$
6	$-$	$-\frac{1}{8^3}\sin 8x$
0		$\frac{1}{8^4}\cos 8x$

$$\frac{x^3}{8}\sin 8x + \frac{3x^2}{8^2}\cos 8x - \frac{6x}{8^3}\sin 8x - \frac{6}{8^4}\cos 8x$$

6.5 Logistic Growth

$$\frac{dP}{dt} = kP(M-P)$$

$$\int \frac{dP}{P(m-P)} = \int k\,dt$$

$$= kt + C$$

$$\cancel{\frac{1}{P(m-P)} = \frac{A(m-P)}{P(m-P)} + \frac{BP}{(m-P)(P)}}$$

$$1 = A(m-P) + BP$$

$P = 0$ $\quad 1 = Am$

$A = \frac{1}{m}$

$P = m$

$1 = Bm$

$B = \frac{1}{m}$

$$\int \frac{\frac{1}{m}}{P} + \frac{\frac{1}{m}}{m-P}\,dP = kt + C$$

$$m\frac{1}{\cancel{m}}\ln P \cancel{+} m\frac{1}{\cancel{m}}\frac{\ln(m-P)}{-1} = mkt + C$$

$$\ln P - \ln(m-P) = mkt + C$$

$$\ln\left(\frac{P}{m-P}\right) = mkt + C$$

$$e^{mkt+C} = \frac{P}{m-P}$$

$$Ce^{mkt} = \frac{P}{m-P}$$

$$\frac{1}{Ce^{mkt}} = \frac{m-P}{P}$$

$$\frac{Ce^{mkt}}{Ce^{mkt}} + \frac{1}{Ce^{mkt}} = \frac{m}{P} - \cancel{\frac{P}{P}}$$

$$\frac{Ce^{mkt}+1}{Ce^{mkt}} = \frac{m}{P}$$

$$\frac{\frac{1}{Ce^{mkt}}Ce^{mkt}}{\frac{1}{Ce^{mkt}}(Ce^{mkt}+1)} = \frac{P}{m}$$

$$\frac{1}{1+\frac{1}{Ce^{mkt}}}$$

$$\frac{1}{1+Ae^{-mkt}} = \frac{P}{m}$$

$$\boxed{\frac{m}{1+Ae^{-mkt}} = P}$$

$$A = \frac{m-P_0}{P_0}$$

$\ln(5-x)$

$\frac{1}{5-x}\cdot$

solve the differential equation: $\frac{dP}{dt}=kP\left(M-P\right)$

Sixty one moose were introduced to the upper peninsula in Michigan. The growth rate is given below. Solve for P

$$\frac{dP}{dt} = .0003P(1000 - P)$$

$$A = \frac{M - P_0}{P_0}$$

$$P = \frac{1000}{1 + 15.393e^{-1\cancel{0}\cancel{0}\cancel{0}(.0003)t}}$$

$$A = \frac{1000 - 61}{61}$$

$$P = \frac{1000}{1 + 15.393e^{-.3t}}$$

General solution to the logistic differential equation:

$$P = \frac{M}{1 + Ae^{-(Mk)t}} \qquad A = \frac{M - P_0}{P_0}$$

$$\frac{dP}{dt} = kP\left(M - P\right)$$

$$P\left(2 - \frac{P}{5000}\right)$$

$$\frac{dP}{dt} = \frac{1}{5000}P\left(10000 - P\right)$$