41

$\frac{dy}{dt}$ $y=y_0e^{kt}$ $\frac{dp}{dh}$ $p=p_0e^{kh}$

$h=0$
$p=1013$

$1013=P_0e^{k(0)}$

$P=1013e^{-.121h}$

$P=1013e^{kh}$

$\frac{90}{1013}=\cancel{1013}e^{20k}$

$\frac{\ln\left(\frac{90}{1013}\right)}{20}=\cancel{20}k$

$K=-.121$

33.

$T-T_s=79.466(\quad)^t$

$T=79.466(\quad)^t+10$

$T-10=79.466(\quad)^t$

$2=79.466(\quad)^t$

32. 60° above room temp.

70° " " " 20 min ago.

$$(T - T_s) = (T_0 - T_s)e^{kt}$$

$$60^\circ = 70^\circ e^{k(20)}$$

$$? = 60e^{-.0077(15)}$$

4. $\frac{dy}{dx} = 2xy$

$$\int \frac{dy}{y} = \int 2x \; dx$$

$x = 0$
$y = 3$

$$\ln|y| = x^2 + C$$

$$\ln 3 = C$$

$$\ln|y| = x^2 + \ln 3$$

$$e^{x^2 + \ln 3} = y$$

$$e^{x^2} \cdot \cancel{e}^{\cancel{\ln} 3} = y$$

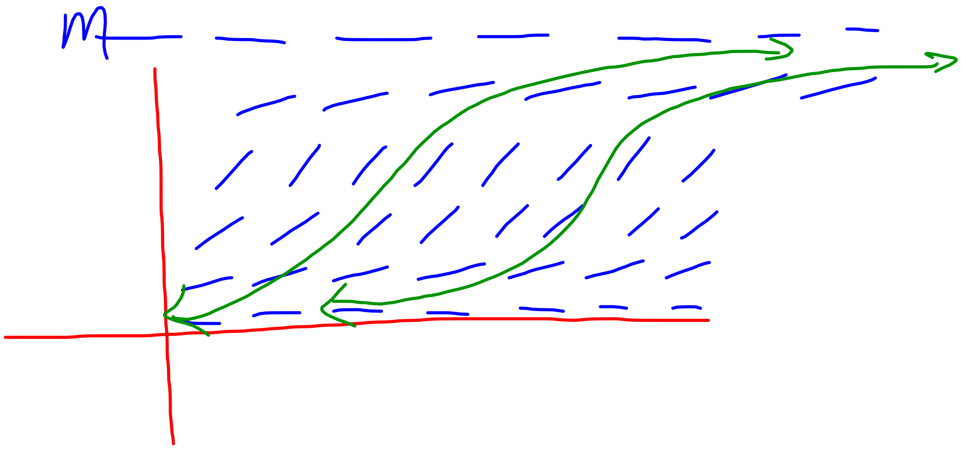
$$3e^{x^2} = y$$

6.5 Logistic Growth

80(100 − 80) population carrying capacity

$$\frac{dP}{dt} = kP\left(M - P\right)$$

Use the differential equation to construct a slopefield



The growth rate of a population P of bears in a newly established wildlife preserve is modeled by the differential equation

$$\frac{dP}{dt} = .008P(100 - P)$$ where t is measured in years.

(a) What is the carrying capacity for bears in this preserve? 100

(b) What is the bear population when the population is growing the fastest? 50

(c) What is the rate of change of the population when it is growing the fastest.

$20 \ \frac{\text{bears}}{\text{yr.}}$

solve the differential equation: $\frac{dP}{dt} = kP(M-P)$

$$\int \frac{dP}{P(M-P)} = \int k \, dt$$

$$kt + C$$

Partial fractions:

$$f(x) = \frac{3}{x(x+1)} = \frac{A(x+1)}{x(x+1)} + \frac{B(x)}{(x+1)(x)}$$

$$3 = A(x+1) + Bx$$

$x = -1$ $\quad 3 = \cancel{A(0)} + B(-1)$

$$B = -3$$

$x = 0$ $\quad 3 = A(1) + B(0)$

$$A = 3$$

$$\int \frac{3}{x(x+1)} = \int \frac{3}{x} + \int \frac{-3}{x+1}$$

$$3\int \frac{1}{x} dx - 3\int \frac{1}{x+1} dx$$

$$3 \ln|x| - 3 \ln|x+1| + C$$

$$f(x)=\frac{1}{x^2-x-6}=\frac{A}{x-3}+\frac{B}{x+2}$$

$$= \frac{1}{5(x-3)}$$

$$\int\frac{1}{x^2-x-6} = \int\frac{\frac{1}{5}}{x-3} + \int\frac{-\frac{1}{5}}{x+2}$$

$$= \frac{1}{5}\int\frac{1}{x-3}\,dx - \frac{1}{5}\int\frac{1}{x+2}\,dx$$

$$\int\frac{1}{x^2-x-6}\,dx = \frac{1}{5}\ln|x-3| - \frac{1}{5}\ln|x+2| + C$$

$$\ln\left|\frac{x-3}{x+2}\right|^{\frac{1}{5}} + C$$

$$f(x)=\frac{5x+4}{x^2-2x-8}$$

$$\frac{5x+4}{x^2-2x-8} = \frac{A(x+2)}{x-4} + \frac{B(x-4)}{x+2}$$

$$5x+4 = A(x+2) + B(x-4)$$

$x=-2$ $\quad -6 = B(-6)$

$B=1$

$x=4$ $\quad 24 = A(6)$

$A=4$

$$\int\frac{5x+4}{(x-4)(x+2)}\,dx = \int\frac{4}{x-4}\,dx + \int\frac{1}{x+2}\,dx$$

$$= 4\ln|x-4| + \ln|x+2| + C$$

$$f(x) = \frac{x - 13}{2x^2 - 7x + 3}$$

Sixty one moose were introduced to the upper peninsula in Michigan. The growth rate is given below. Solve for P

$$\frac{dP}{dt} = .0003P(1000 - P)$$

General solution to the logistic differential equation:

$$\frac{dP}{dt} = kP\left(M - P\right) \qquad P = \frac{M}{1 + Ae^{-(Mk)t}} \qquad A = \frac{M - P_0}{P_0}$$