6.3

21. $\int x^4 e^{-x}\,dx$

u		dv
x^4		$e^{-x}\,dx$
$4x^3$	+	$-e^{-x}$
$12x^2$	−	e^{-x}
$24x$	+	$-e^{-x}$
24	−	e^{-x}
0	+	$-e^{-x}$

$$-x^4e^{-x} - 4x^3e^{-x} - 12x^2e^{-x} - 24xe^{-x} - 24e^{-x} + C$$

47. $$\int x^n \cos x\,dx = x^n \sin x - n\int x^{n-1}\sin x\,dx$$

$u = x^n \qquad dv = \cos x\,dx$

$du = nx^{n-1}\,dx \qquad v = \sin x$

$$= x^n \sin x - \int n x^{n-1} \sin x\,dx$$

43

$$\int \sin\sqrt{x}\,dx \qquad u = x^{\frac{1}{2}}$$

$$\frac{2\,du}{x^{-\frac{1}{2}}} = \frac{1}{2}x^{-\frac{1}{2}}dx$$

$$\int 2x^{\frac{1}{2}}\sin u\,du$$

$$\int 2u\sin u\,du$$

$$2\int u\sin u\,du \qquad w = u \quad dv = \sin u\,du \qquad dw = du \quad v = -\cos u$$

$$2\left(-u\cos u + \int +\cos u\,du\right)$$

$$2\left(-u\cos u + \sin u\right) + C$$

$$2\left(-\sqrt{x}\cos\sqrt{x} + \sin\sqrt{x}\right) + C$$

Circular Integration by Parts

$$\int e^x \cos x\,dx \qquad u = e^x \quad dv = \cos x\,dx \qquad du = e^x dx \quad v = \sin x$$

$$= e^x \sin x - \int e^x \sin x\,dx \qquad u = e^x \quad dv = \sin x\,dx \qquad du = e^x du \quad v = -\cos x$$

$$e^x \sin x - \left(-e^x \cos x - \int -e^x \cos x\,dx\right)$$

$$\int e^x \cos x\,dx = e^x \sin x + e^x \cos x - \int e^x \cos x\,dx$$

$$+\int e^x \cos x\,dx \qquad\qquad +\int e^x \cos x\,dx$$

$$\frac{\cancel{2}\int e^x \cos x\,dx}{\cancel{2}} = \frac{e^x \sin x}{2} + \frac{e^x \cos x}{2}$$

$$\int e^x \cos x\,dx = \frac{e^x \sin x}{2} + \frac{e^x \cos x}{2} + C$$

15. 1000 8.6% Doub T. \$ in 30 yr.

$$\frac{\ln 2}{.086} =$$

$$y = 1000e^{30(.086)}$$

25.

$$y = y_0 e^{-.18t}$$

$$.9\cancel{y_0} = \cancel{y_0} e^{-.18t}$$

$$.9 = e^{-.18t}$$

$$\frac{\ln(.9)}{-.18} = \cancel{-.18}t$$

24.

3 h 10,000
5 h 40,000

$y_0 = 1250$

$\frac{10000}{e^{3k}} = \frac{y_0 e^{3k}}{e^{3k}}$

$y_0 = \frac{10000}{e^{3k}}$

$40000 = y_0 e^{5k}$

$40000 = \frac{10000}{e^{3k}} \cdot e^{5k}$

$\cancel{40000} = \cancel{10000} e^{2k}$

$4 = e^{2k}$

$\frac{\ln 4}{2} = \frac{\cancel{2}k}{\cancel{2}}$

$k = .693$

23.

$y_0 = 1$

double time: .5 hr.

$k = \frac{\ln 2}{.5}$

$y = 1e^{(\ )(24)}$

2.8×10^{14}

6.4b Exponential Growth and Decay

Newton's Law of Cooling: The rate at which an object's temperature is changing is directly proportional to the difference between its temperature and the temperature of the surrounding medium.

$$\frac{dT}{dt} = k(T - T_s) \qquad \frac{dy}{dt} = ky$$

$$y = y_0 e^{kt}$$

$$(T - T_s) = (T_0 - T_s)e^{kt}$$

T = temp.

T_s = temp. surrounding

T_0 = initial temp.

t = time

ex 6: A hard boiled egg at 98 degrees Celsius is put in a pan under running 18 degree water to cool. After 5 minutes, the egg's temperature is found to be 38 degrees. How much longer will it take the egg to reach 20 degrees?

$$(38 - 18) = (98 - 18)e^{5k}$$

$$\frac{20}{80} = \cancel{80}e^{5k}$$

$$\frac{1}{4} = e^{5k}$$

$$\frac{\ln\left(\frac{1}{4}\right)}{5} = \cancel{5}k$$

$$k = -.277$$

$$(20-18) = (98-18)e^{t(-.277)}$$

$$\frac{2}{80} = \cancel{80}e^{-.277t}$$

$$\frac{1}{40} = e^{-.277t}$$

$$\frac{\ln\left(\frac{1}{40}\right)}{-.277} = -.277t$$

$$t = 13.305$$

ex 6: A hard boiled egg at 98 degrees Celsius is put in a pan under running 18 degree water to cool. After 5 minutes, the egg's temperature is found to be 38 degrees. How much longer will it take the egg to reach 20 degrees?

$$(38-18) = (98-18)e^{5k}$$

$$\frac{20}{80} = \cancel{80}e^{5k}$$

$$\frac{1}{4} = e^{5k}$$

$$\frac{\ln\left(\frac{1}{4}\right)}{5} = \cancel{5}k$$

$$k = -\frac{\ln 4}{5} = -.277$$

$$(20-18) = (38-18)e^{(-.277)t}$$

$$\frac{2}{20} = \frac{\cancel{20}}{20}e^{-.277t}$$

$$\frac{1}{10} = e^{-.277t}$$

$$\frac{\ln\left(\frac{1}{10}\right)}{-.277} = \cancel{-.277}t$$

$$t = 8.305$$