22. $\displaystyle\int \frac{9t^2\,dt}{\sqrt{1-t^3}}$

$u = 1 - t^3$

$du = -3t^2\,dt$

$$\int \frac{\overset{-3}{\cancel{9}}\cancel{t^2}}{u^{\frac{1}{2}}} \frac{du}{\cancel{-3t^2}}$$

$$-3\int u^{-\frac{1}{2}}\,du$$

53. $\displaystyle\int_0^3 \sqrt{y+1}\,dy$

$u = y+1$

$du = 1\,dy$

$$\int_1^4 u^{\frac{1}{2}}\,du$$

$$\frac{2}{3}u^{\frac{3}{2}}\Big|_1^4 = \frac{2}{3}(8) - \frac{2}{3}$$

$$\frac{14}{3}$$

9. $\int y \ln y \, dy$

$u = \ln y \qquad dv = y\,dy$

$du = \frac{1}{y}\,dy \qquad v = \frac{y^2}{2}$

$\frac{y^2}{2}\ln y - \frac{1}{2}\int \frac{y^{\cancel{2}}}{2} \cdot \frac{1}{\cancel{y}}\,dy$

$\frac{y^2 \ln y}{2} - \frac{y^2}{4} + C$

35.

$y = 2e^{-t}\cos t \qquad t \geq 0$

$\frac{1}{2\pi}\int_0^{2\pi} 2e^{-t}\cos t \, dt$

47.

$$\int x^n \cos x\,dx = x^n \sin x - n\int x^{n-1}\sin x\,dx$$

$u = x^n$ $dv = \cos x\,dx$

$du = n x^{n-1}dx$ $v = \sin x$

$$x^n \sin x - \int \sin x \cdot n x^{n-1}\,dx$$

QR

9.

$$\int \frac{dy}{dx} = \int x + \sin x \qquad y(0) = 2$$

$$y = \frac{x^2}{2} - \cos x + C$$

$$2 = 0 - \cos(0) + C$$

$$2 = -1 + C$$

$$3 = C$$

13.

$$\int \frac{du}{\cancel{dx}} = \int x \sec^2 x \, dx \qquad u=1 \quad x=0$$

$$u = x \qquad dv = \sec^2 x \, dx$$

$$du = dx \qquad v = \tan x$$

$$x\tan x - \int \overset{\frac{\sin x}{\cos x}}{\tan x} \, dx$$

$$u = \cos x$$

$$du = -\sin x \, dx$$

$$x\tan x - \int \frac{\cancel{\sin x}}{u} \cdot \frac{du}{\cancel{-\sin x}}$$

$$x\tan x + \ln|u|$$

$$u = x\tan x + \ln|\cos x| + C$$

$$1 = 0 + \underbrace{\ln 1}_{0} + C$$

6.4a Exponential Growth and Decay

separation of variables:

find a general solution $\dfrac{dy}{dx} = \dfrac{2x}{y}$

$$\int y \cdot dy = \int 2x \, dx$$

$$\frac{y^2}{2} = x^2 + C$$

$$\frac{dy}{dx} = x(1+y)$$

$(1+y)^{-1}$

$$\frac{dy}{1+y} = x\ dx$$

$$\frac{1}{1+y}\,dy = x\ dx$$

$$\ln|1+y| = \frac{x^2}{2} + C$$

Law of exponential change:

If y changes at a rate proportional to the amount present:

where k is the growth or decay constant

if $\frac{dy}{dt} = ky$ then $y = y_0 e^{kt}$

$$\frac{dy}{y} = k\ dt$$

$$\ln|y| = kt + C$$

$y > 0$

$$e^{kt+C} = y$$

$x^2 \cdot x^3 = x^{2+3}$

$$e^{kt} \cdot e^{c} = y$$

$$Ce^{kt} = y$$

Suppose you deposit $600 in an account that pays 5.2% annual interest compounded continuously. How much do you have 8 years later?

$$\frac{5.2}{100}$$

$$y = 600e^{.052(8)}$$

$$\$909.53$$

How long will it take to double?

$$1200 = 600e^{.052t}$$

$$2 = e^{.052t}$$

$$\frac{\ln 2}{.052} = \frac{\cancel{.052}t}{}$$

$$13.329 = t$$

Doubling Time:

$$\frac{\ln 2}{k} = t$$

How long will it take $5000 to double at 7%, compounded continously?

$$t = \frac{\ln 2}{.07} = 9.902$$

Radioactive decay and half-life:

$$\frac{dy}{dt} = -ky \qquad y = y_0 e^{-kt}$$

Find the half-life:

$$\frac{1}{2}\cancel{y_0} = \cancel{y_0} e^{-kt}$$

$$\frac{1}{2} = e^{-kt}$$

$$\ln\left(\frac{1}{2}\right) = -kt$$

$$\cancel{\ln 1} + \ln 2 = +kt$$

$$\frac{\ln 2}{k} = t$$

The half-life of carbon 14 is about 5700 years. Find the age of a sample in which 10% of the radioactive nuclei originally present have decayed.

$$y_0 \qquad y = .90\, y_0$$

$$\frac{\ln 2}{k} = \frac{t}{1}$$

$$\frac{\ln 2}{t} = k$$

$$\frac{\ln 2}{5700} = k$$

$$.90\cancel{y_0} = \cancel{y_0} e^{-(.00012)t}$$

$$.9 = e^{-.00012t}$$

$$\frac{\ln(.9)}{-.00012} = t$$

$$t = 866.418$$

Suppose an experimental population of fruit flies increases according to the law of exponential growth. There were 100 flies after the second day of the experiment and 300 flies after the fourth day. Approximately how many flies were in the original population?