

2000  
# 2

$\frac{1}{2}(10)(7+10)$

a.  $y = \frac{10}{3}x$

b. A  $\frac{10}{3}$       B  $\frac{24t}{2t+3}$

$\frac{(2t+3)(24) - (24t)(2)}{(2t+3)^2}$

$\frac{72}{(2t+3)^2} \Big|_{t=2} = \frac{72}{49}$

$\int_0^{10} \frac{24t}{2t+3} dt = 83.336$  meters

27.

$\int \sqrt{\tan x} \sec^2 x dx$        $u = \tan x$   
 $du = \sec^2 x dx$

$\int u^{\frac{1}{2}} du$

$\frac{2}{3} u^{\frac{3}{2}} + C$

$\frac{2}{3} (\tan x)^{\frac{3}{2}} + C$

$$3. \int t^2 - \frac{t^{-2}}{t^2} dt$$

$$\frac{t^3}{3} - \frac{t^{-1}}{-1} + C$$

$$33. \int \frac{\ln^6 x}{x} dx = \int \frac{(\ln x)^6}{x} dx$$

$$u = \ln x$$

$$x \cdot du = \frac{1}{x} dx$$

$$\int \frac{u^6}{x} \cdot x du$$

$$\int u^6 du$$

$$\frac{u^7}{7} + C$$

$$\frac{(\ln x)^7}{7} + C$$

37.

$$\int \frac{\sin(2t+1)}{\cos^2(2t+1)} dt \quad u = \cos(2t+1)$$

$$\frac{du}{-2\sin(2t+1)} = -\sin(2t+1) \cdot 2 dt$$

$$\int \frac{\cancel{\sin(2t+1)}}{u^2} \cdot \frac{du}{-2\cancel{\sin(2t+1)}}$$

$$-\frac{1}{2} \int \frac{u^{-2}}{u^2} du$$

$$+\frac{1}{2} \left( \frac{u^{-1}}{-1} \right) + C$$

$$\frac{1}{2u} + C$$

$$\frac{1}{2 \cos(2t+1)} + C$$

21.

$$\int \frac{dx}{x^2+9} \quad 3u = \frac{x}{3}$$

$$3 \cdot du = \frac{1}{3} dx$$

$$\int \frac{3 du}{(3u)^2 + 9}$$

$$\int \frac{3}{9u^2 + 9} du$$

$$\int \frac{\cancel{3}}{3 \cdot 9(u^2+1)} du$$

$$\frac{1}{3} \int \frac{1}{u^2+1} du$$

$$\frac{1}{3} \tan^{-1} u + C$$

$$\frac{1}{3} \tan^{-1} \left( \frac{x}{3} \right) + C$$

$$39. \int \frac{dx}{x \ln x}$$

$$u = \ln x$$

$$x \cdot du = \frac{1}{x} dx$$

$$\int \frac{\cancel{x} du}{\cancel{x} u}$$

$$\int \frac{1}{u} du$$

$$\ln |u| + C$$

$$\ln |\ln x| + C$$



$$\int \frac{dx}{\cos^2 2x}$$

$$\int \sec^2 2x dx$$

$$u = 2x$$

$$\frac{du}{2} = \frac{1}{2} dx$$

$$\frac{\tan 2x}{2} + C$$

$$\int \cot^2(3x) dx = \int \csc^2 3x - 1 dx$$

$$\int \frac{\cancel{\cos^2 3x}}{\cancel{\sin^2 3x}} dx$$

$$- \frac{\cot 3x}{3} - x + C$$

$$\int \cos^3 x dx = \int \cos x \cos^2 x dx$$

$$\int \cos x (1 - \sin^2 x) dx$$

$$u = \sin x \\ du = \cos x dx$$

$$\int (1 - u^2) du$$

$$u - \frac{u^3}{3} + C$$

$$\sin x - \frac{\sin^3 x}{3} + C$$

Definite Integrals:

$\int_0^{\frac{\pi}{3}} \tan x \sec^2(x) dx$

$u = \tan x$   
 $du = \sec^2 x dx$

$\int_0^{\sqrt{3}} u du$

$\frac{u^2}{2} \Big|_0^{\sqrt{3}} = \frac{3}{2} - 0 = \frac{3}{2}$

$\frac{(\tan x)^2}{2} \Big|_0^{\frac{\pi}{3}} = \frac{(\tan \frac{\pi}{3})^2}{2} - \frac{(\tan 0)^2}{2}$

$\frac{3}{2} - 0 = \frac{3}{2}$

$\int_0^1 \frac{x}{x^2 - 4} dx$

$u = x^2 - 4$   
 $du = 2x dx$

$\int_{-4}^{-3} \frac{\cancel{x}}{u} \frac{du}{2\cancel{x}}$

$\frac{1}{2} \int \frac{1}{u} du$

$\frac{1}{2} \ln|u| \Big|_{-4}^{-3} = \frac{1}{2} (\ln 3 - \ln 4)$

$\frac{1}{2} \ln\left(\frac{3}{4}\right)$

$\frac{1}{2} \ln|x^2 - 4| \Big|_0^1 = \frac{1}{2} (\ln 3 - \ln 4)$

19.

$$\int \sec 2x \tan 2x \, dx \quad \begin{array}{l} u = 2x \\ du = 2 \, dx \end{array}$$

$$\int \sec u \tan u \frac{du}{2}$$

$$\frac{1}{2} \int \sec u \tan u \, du$$

$$\frac{1}{2} \sec u + C$$

$$\frac{1}{2} \sec 2x + C$$

21.

$$\int \frac{dx}{x^2+9}$$

$$\begin{array}{l} 3 u = \frac{x}{3} \\ 3 du = \frac{1}{3} dx \end{array}$$

$$\int \frac{3 \, du}{(3u)^2+9}$$

$$\int \frac{3 \, du}{9u^2+9}$$

$$\int \frac{\cancel{3} \, du}{9(u^2+1)}$$

$$\frac{1}{3} \int \frac{1}{u^2+1} \, du$$

$$\frac{1}{3} \tan^{-1} u + C$$

$$\frac{1}{3} \tan^{-1} \left( \frac{x}{3} \right) + C$$

37.

$$\int \frac{\sin(2t+1)}{\cos^2(2t+1)} dt$$

$$u = \cos(2t+1)$$

$$\frac{du}{-2\sin(2t+1)}$$

$$\int \frac{\cancel{\sin(2t+1)}}{u^2} \frac{du}{-2\cancel{\sin(2t+1)}}$$

$$-\frac{1}{2} \int u^{-2} du$$

$$+\frac{1}{2} \left( \frac{u^{-1}}{-1} \right) + C$$

$$\frac{1}{2 \cos(2t+1)} + C$$

39.

$$\int \frac{dx}{x \ln x}$$

$$u = \ln x$$

$$du = \frac{1}{x} dx$$

$$\int \frac{1}{u} du$$

$$\ln u + C$$

$$\ln(\ln x) + C$$



47.

$$\int \sin^3 2x \, dx$$

$\sin^2 2x = 1 - \cos^2 2x$

$$\sin 2x \cdot \sin^2 2x$$

$$\int \sin 2x (1 - \cos^2 2x) \, dx$$

$$u = \cos 2x$$

$$\frac{du}{-2 \sin 2x} = -2 \sin 2x \, dx$$

$$-2 \sin 2x$$

$$\int \cancel{\sin 2x} (1 - u^2) \frac{du}{\cancel{-2 \sin 2x}}$$

$$-\frac{1}{2} \int (1 - u^2) \, du$$

$$-\frac{1}{2} \left( u - \frac{u^3}{3} \right) + C$$

$$\int \sin^3 2x \, dx = -\frac{1}{2} \left( \cos 2x - \frac{\cos^3 2x}{3} \right) + C$$