

Trapezoid Rule:

general rule:

$$T = \frac{h}{2}(y_0 + 2y_1 + 2y_2 + \dots + 2y_{n-1} + y_n)$$

average of LRAM and RRAM

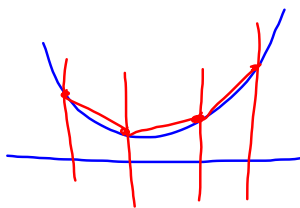
$$LRAM = h(y_0 + y_1 + y_2 + \dots + y_{n-1})$$

$$RRAM = h(y_1 + y_2 + \dots + y_{n-1} + y_n)$$

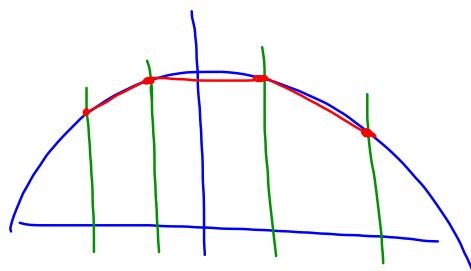
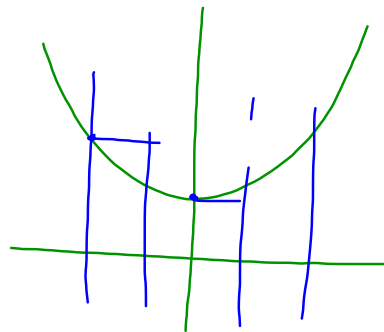
$$T = \frac{L + R}{2}$$

Concavity in estimates:

Trapezoids:



Over Est.



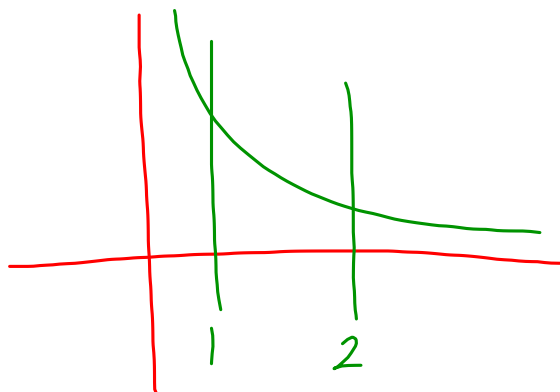
Under Est.

Estimate  $\int_1^2 \frac{1}{x} dx$  with 10 trapezoids

how to use calculator

over or under estimate?

because the curve is concave up.  
.69377



Calculate the area of  $\int_2^4 2x^2 dx$  using 4 trapezoids



$$A \approx \frac{1}{2} \left( \frac{h}{2} \right) \left( 8 + 32 + 2 \left( \frac{25}{2} + 18 + \frac{49}{2} \right) \right)$$

$$\frac{1}{4} (8 + 32 + 25 + 36 + 49)$$

$$\frac{1}{4} (150) = 37.5 \text{ or } \frac{75}{2}$$

Would you expect this to be an over or under estimate?

over est. because curve is  
concave up

An observer measures the outside temperature every hour from noon until midnight, recording the temperature in the following table:

time	noon	1	2	3	4	5	6	7	8	9	10	11	mid
temp	63	65	66	68	70	69	68	68	65	64	62	58	55

What was the average temperature for the 12-hour period?

$$\frac{A}{12} \approx \frac{1}{2} (1)(63+55+2(65+66+68+70+69+68+68+65+64+62+58))$$

$$\text{Avg.} = 65.166$$

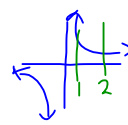
Error Bound for the Trapezoid rule:

$$|E| \leq \frac{b-a}{12} h^2 M_{f''} \quad M_{f''} \text{ is the max value of } f''$$

Estimate the error in approximating  $\int_1^2 \frac{1}{x} dx$  with 10 trapezoids.

$$f = \frac{1}{x}$$

$$f' = -x^{-2}$$

$$M_{f''} = f'' = 2x^{-3} = \frac{2}{x^3} \Big|_{x=1} = 2$$


$$|E| \leq \frac{2-1}{12} (.1)^2 2$$

$$|E| \leq \frac{1}{12} \cdot \frac{1}{100} \cdot 2$$

$$|E| \leq \frac{1}{600}$$