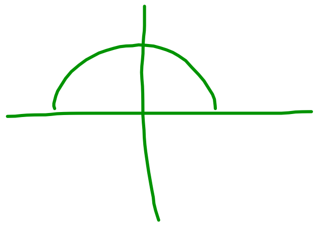
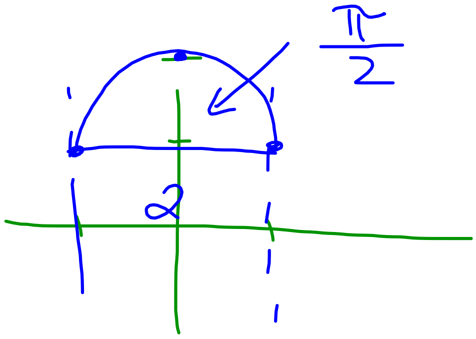
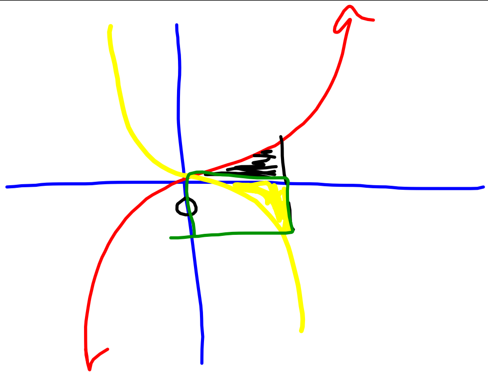
15. $\int_{-3}^{3} \sqrt{9-x^2}\, dx$



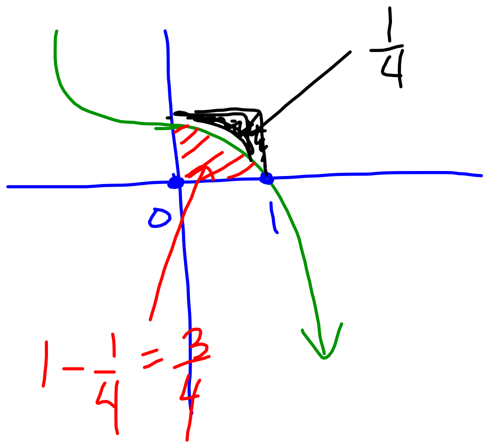
20. $\int_{-1}^{1} 1+\sqrt{1-x^2}$



51. $\int_0^1 x^3\, dx = \frac{1}{4}$



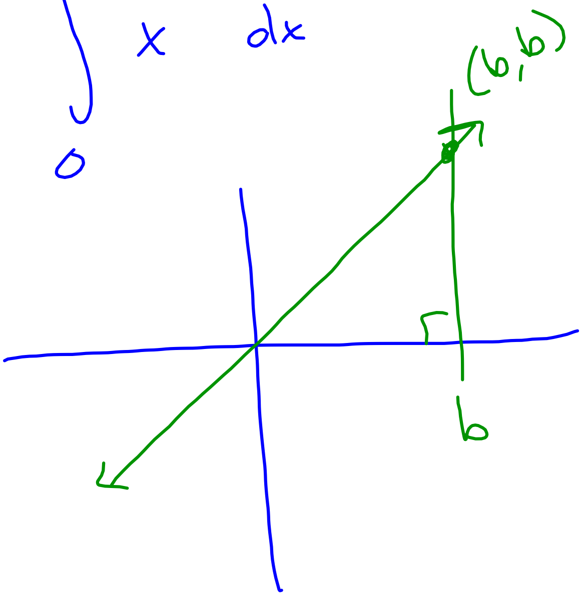
$\int_0^1 (1-x^3)\, dx$



23.

$$\int_0^b x \quad dx$$

$b > 0$



$A = \frac{1}{2} b \cdot h$

$A = \frac{1}{2} b \cdot b$

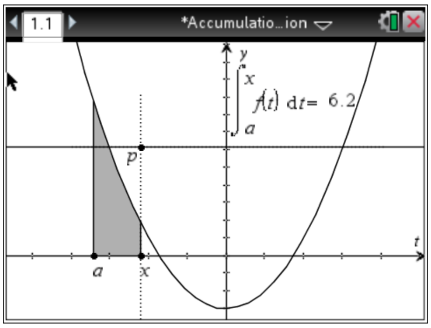
29.

$$\int_8^{11} 87 \, dx = 261$$

5.3 Definite Integrals and Antiderivatives

A function defined by a definite integral:

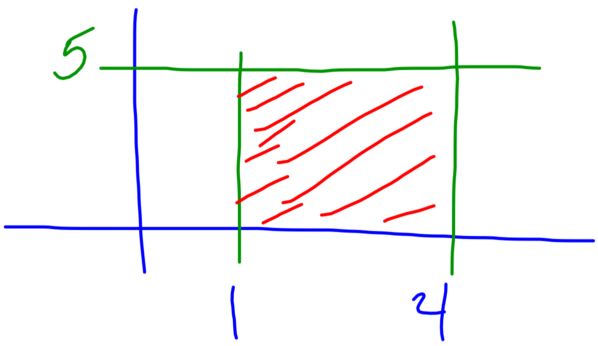
$$Area = \int_a^x f(t)\,dt$$



What is the relationship between A(x) and f(x)?

A(x) is antiderivative of f(x)

$$F(x) = \int_a^x f(t)\,dt$$



$$\int_4^1 5\,dx = 5(1-4) = 5(-3) = -15$$

Evaluate the following integrals and look for patterns:

$$\int_0^x (t)\,dt = \frac{x^2}{2}$$

$$\int_0^x 5t^2\,dt = 5\int_0^x t^2\,dt$$

$$\int_0^x (t^2)\,dt = \frac{x^3}{3}$$

$$\int_0^x (t^3)\,dt = \frac{x^4}{4}$$

Evaluate the following integrals and look for patterns:

$$\int_a^x (t)\,dt = \frac{x^2}{2} - \frac{a^2}{2}$$

$$\frac{t^2}{2}$$

$$\int_a^x (t^2)\,dt = \frac{x^3}{3} - \frac{a^3}{3}$$

$$\int_a^x (t^3)\,dt = \frac{x^4}{4} - \frac{a^4}{4}$$

$$\int_a^x f(t)\,dt = F(x) - F(a)$$

$$F'(t) = f(t)$$

Evaluate the following integrals by hand:

$$\int_0^{\pi} \sin(x)\,dx = -\cos x \Big|_0^{\pi} = -\cos\pi - (-\cos 0)$$

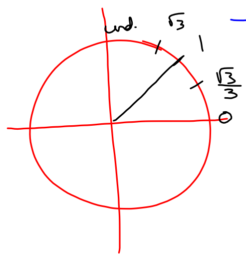
$$1 + 1 = 2$$

$$\int_2^3 \left(x^3 + x - 1\right) dx = \left(\frac{x^4}{4} + \frac{x^2}{2} - x\right)\Big|_2^3$$

$$\left(\frac{81}{4} + \frac{9}{2} - 3\right) - \left(\frac{16}{4} + \frac{4}{2} - 2\right)$$

$$\frac{87}{4} - 4 = \frac{71}{4}$$

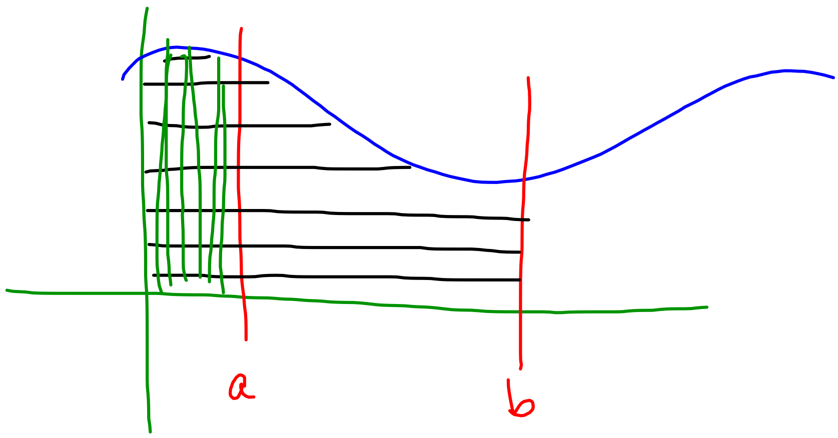
$$\int_0^1 \left(\frac{1}{1+x^2}\right) dx = \tan^{-1} x \Big|_0^1 =$$



$$\tan^{-1} 1 - \tan^{-1} 0$$

$$\frac{\pi}{4} - 0 = \frac{\pi}{4}$$

$$F(b) - F(a)$$



Rules for Definite Integrals

$\int_a^a f(x)dx = 0$

$\int_a^b f(x)dx = -\int_b^a f(x)dx$ $\qquad \int_4^1 5\,dx = -\int_1^4 5\,dx$

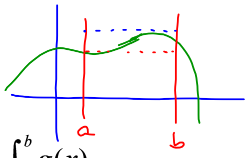
$\int_a^b (f(x) \pm g(x))dx = \int_a^b f(x)dx \pm \int_a^b g(x)dx$

$\int_a^b -f(x)dx = -\int_a^b f(x)dx = \int_b^a f(x)dx$ $\qquad \int -5x^2 dx$

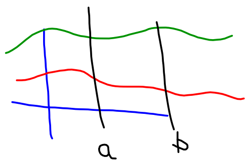
$\int_a^b c \cdot f(x)dx = c\int_a^b f(x)dx$ $\qquad -5\int x^2 dx$

* $\int_a^b f(x)dx + \int_b^c f(x)dx = \int_a^c f(x)dx$

$f_{\min}(b-a) \le \int_a^b f(x)dx \le f_{\max}(b-a)$



if $f(x) > g(x)$ then $\int_a^b f(x) > \int_a^b g(x)$



Ex.

$\int_{-4}^{3} f(x)dx = 9 \qquad \int_{3}^{5} f(x)dx = -11 \qquad \int_{-4}^{3} h(x)dx = 14$

a. $\int_5^3 f(x)dx =$ $-\int_3^5 f(x)dx = -(-11) = 11$

b. $\int_{-4}^{5} f(x)dx =$ $9 + -11 = -2$

c. $\int_{-4}^{3} (3f(x) - 4h(x))dx =$

$3(9) - 4(14) = -29$

d. $\int_3^4 f(x)dx =$ not enough info.