

Warm-Up

What does the following limit mean?

$$\lim_{n \rightarrow \infty} \sum_{i=1}^n (5(c_i)^2 - 2c_i + 7)\Delta x$$

h w

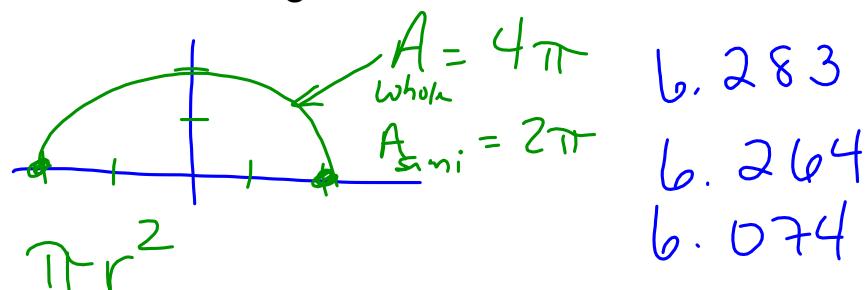
What is the equation for the curve you are finding the area under?

$$5x^2 - 2x + 7$$

5.2 Definite Integrals

Estimate the area under the graph $f(x) = \sqrt{4 - x^2}$

using various rectangular methods with $n = 10, 50, 100, 500$



What happens as n approaches infinity? more accurate

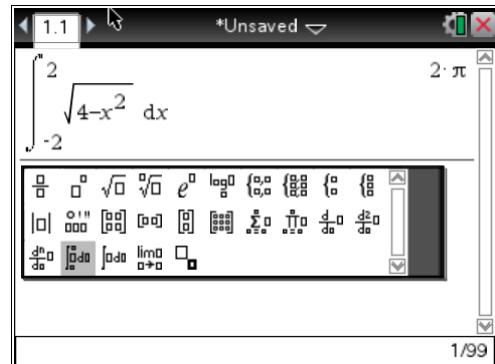
definition of a definite integral as a limit of a Riemann Sum

$$\lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i) \Delta x = \int_a^b f(x) dx$$

upper limit **definite integral**
 lower limit **integrand**
dx variable of integration

if a function is continuous on a given interval then the definite integral exists

$$\int_{-2}^2 \sqrt{4 - x^2} dx =$$



Ex.
express the limit as a definite integral:

$$\lim_{n \rightarrow \infty} \sum_{i=1}^n (4(c_i)^3 - 3c_i + 2)\Delta x \quad [1,6]$$

$$\int_1^6 (4x^3 - 3x + 2) dx$$

Definite Integrals on the Calculator

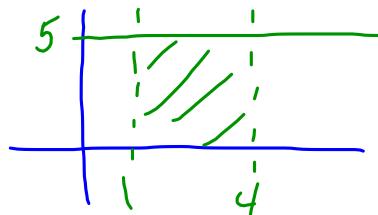
Nspire CAS/updated OS

others

$$\int_{-}^{-} dx$$

NINT(function,variable,lower,upper)

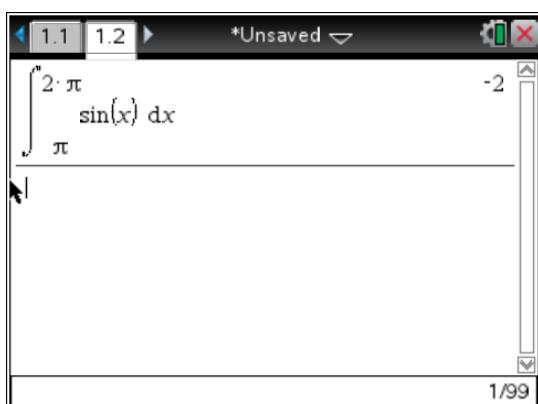
$$\int_1^4 5 dx = 15$$



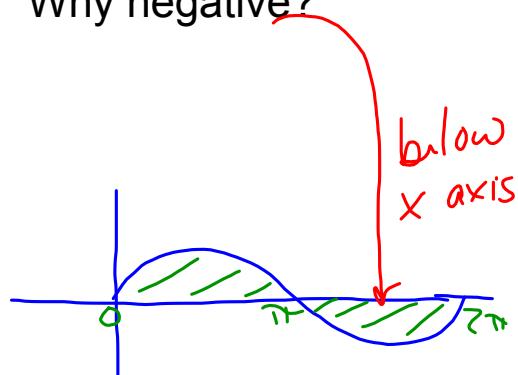
$$\text{so } \int_a^b f(x) dx = \int_a^b c dx = c(b-a)$$

$$\int_{-2}^6 7 dx = 7(6 - (-2)) = 56$$

Definite Integrals and Signed Area



Why negative?



net area vs. total area
displacement

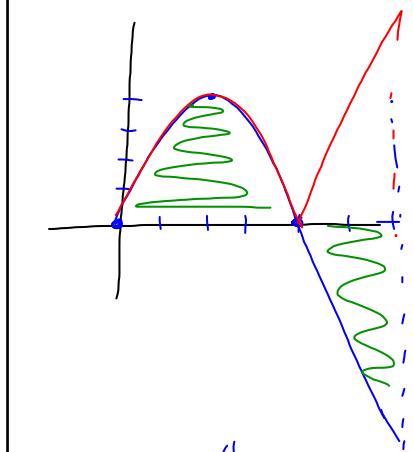
$$\int_0^{2\pi} \sin x = 0$$

$$\int_a^b f(x) dx = \text{displacement}$$

Find the total area bounded by $y = 4x - x^2$ on $[0, 6]$

Calc

$$\int_0^6 |4x - x^2| dx$$



Total Dist. Area

$$\begin{aligned}
 &= \int_0^4 (4x - x^2) dx + \left| \int_4^6 (4x - x^2) dx \right| \\
 &= 21.33 \\
 &= \frac{64}{3}
 \end{aligned}$$

Use the graph of the integrand and areas to evaluate the integral.

$$\int_1^2 (-2x + 6) dx = 3$$

$$\frac{h(b_1 + b_2)}{2}$$

$$\frac{1(4+2)}{2} = 3$$

