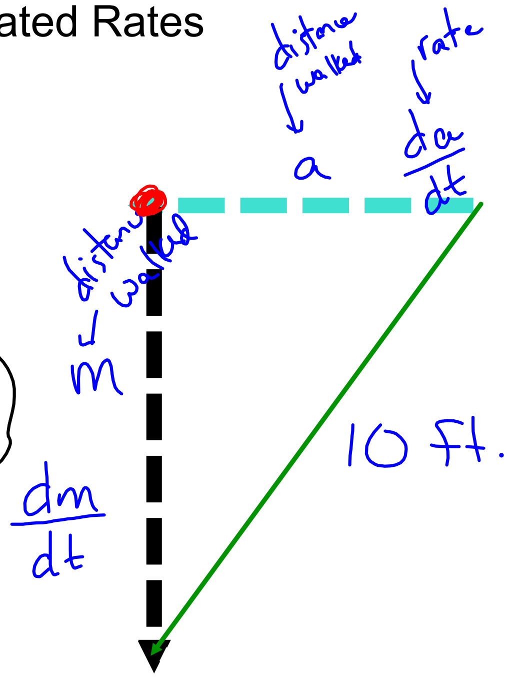
4.6 Related Rates

$$a^2 + m^2 = 10^2$$

$$2a\frac{da}{dt} + 2m\frac{dm}{dt} = 0$$



We have learned to differentiate implicitly defined functions by using the Chain Rule. In this section, we will use the Chain Rule to find the rates of change of two or more variables with respect to time, giving us expressions such as $\dfrac{dy}{dt}, \dfrac{dx}{dt}, \dfrac{dV}{dt}, \dfrac{dr}{dt}$.

Ex. Suppose $y = 5x^2 - 6x + 2$. Find $\dfrac{dy}{dt}$ when $x = 4$, given that $\dfrac{dx}{dt} = 2$ when $x = 4$.

Steps to Use When Solving a Related Rates Word Problem

1. Draw a figure if possible.

2. Assign variables and restate the problem, listing your given information and what you are asked to find. (make sure to notice what changes and what is constant) Notice whether the given rates of change are positive or negative.

3. Find an equation that relates the variables. (may need a 2nd equation)

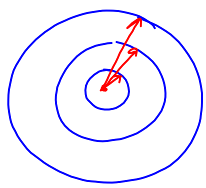
4. Differentiate with respect to time. (implicitly)

5. Substitute the given information, and solve for the unknown derivative. Be sure to include units with your answer.

A pebble is dropped into a calm pond, causing ripples in the shape of concentric circles. The radius of the outer ripple is increasing at a constant rate of 1 ft/sec. When the radius is 4 ft, find the rate at which the area of the disturbed water is changing.

$r = 4 \text{ ft.}$

$\frac{dr}{dt} = 1 \frac{\text{ft.}}{\text{sec.}}$



$\frac{dA}{dt} = ?$

$A = \pi r^2$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

$\frac{dA}{dt} = 2\pi (4 \text{ ft}) \, 1 \frac{\text{ft.}}{\text{sec.}}$

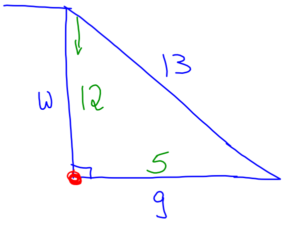
$\frac{dA}{dt} = 8\pi \frac{\text{ft}^2}{\text{sec.}}$

A 13-ft. ladder is leaning against a wall. Suppose that the base of the ladder slides away from the wall at the constant rate of 3 ft/sec. How fast is the top of the ladder sliding down the wall when the base of the ladder is 5 ft. from the wall?

$\frac{dg}{dt} = 3\frac{ft.}{sec.}$

$\frac{dw}{dt} =$

$g = 5$ ft.



$$w^2 + g^2 = 13^2$$

$$2w\frac{dw}{dt} + 2g\frac{dg}{dt} = 0$$

$$2(12ft)\frac{dw}{dt} + 2(5ft)\left(3\frac{ft}{sec}\right) = 0$$

$$24ft.\frac{dw}{dt} + 30\frac{ft^2}{sec} = 0$$

$$-\frac{30}{24}\ \frac{\frac{ft^2}{sec}}{ft.}$$

$$\frac{dw}{dt} = -\frac{5}{4}\ \frac{ft.}{sec.}$$

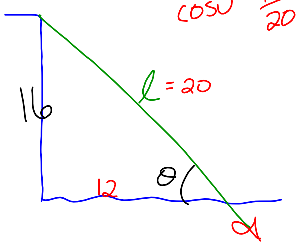
A fish is reeled in at a rate of 2 ft/sec from a bridge that is 16 ft. above the water. At what rate is the angle between the line and the water changing when there are 20 ft of line out?

$\cos\theta = \frac{12}{20}$

$\frac{dl}{dt} = -2\frac{ft.}{sec.}$

$\frac{d\theta}{dt}$

$l = 20$ ft.



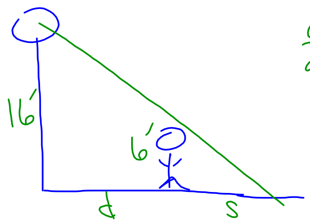
$$\sin\theta = \frac{16}{l}$$

$$\boxed{\cos\theta}\ \frac{d\theta}{dt} = \frac{-16}{l^2}\frac{dl}{dt}$$

$$\cancel{\frac{12}{20}} \cdot \frac{d\theta}{dt} = \frac{+\cancel{16}^{4}}{\cancel{20^2}_{5}}\left(+2\frac{ft.}{sec.}\right) \cdot \frac{\cancel{20}}{\cancel{12}_{3}}$$

$$\frac{d\theta}{dt} = \frac{2}{15}\ \frac{rad.}{sec.}$$

A man 6 ft tall walks at a rate of 5 ft/sec toward a light pole 16 ft. tall. When the man is 10 ft from the base of the light. At what rate if the length of his shadow moving?



$\frac{dd}{dt} = -5 \frac{ft.}{sec.}$

$d = 10$

$\frac{ds}{dt} = ?$

$$\frac{16}{d+s} = \frac{6}{s}$$

$$16s = 6d + 6s$$

$$10s = 6d$$

$$10\frac{ds}{dt} = 6\frac{dd}{dt}$$

$$10\frac{ds}{dt} = 6(-5)$$

$$\frac{ds}{dt} = -3\frac{ft.}{sec.}$$

Water runs out of a conical tank at the constant rate of 2 cubic feet per minute. The radius at the top of the tank is 5 feet, and the height of the tank is 10 feet. How fast is the water level sinking when the water is 4 feet deep?