4.5 Linearizations, Newton's Method and differentials

Newton's Method

locally linear -
$$y - f(x_1) = f'(x_1)(x - x_1)$$

if you have an x-intercept then

solve for x

$f = x^3 + 6x^2 + 9x - 3$

$f' = 3x^2 + 12x + 9 = 0$

$3(x^2 + 4x + 3)$

$3(x+1)(x+3) = 0$

$x = -1, -3$

+ max − min +

−3 −1

$(-3, -3)$ $(-1, -7)$

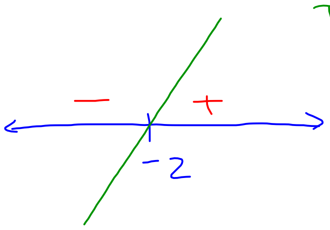
Inflution Pt.

$(-2, -5)$

$f'' = 6x + 12 = 0$

$\frac{6x}{6} = \frac{-12}{6}$

$x = -2$



Concave:

Down: $x < -2$

Up: $x > 2$

Linearizations:

because of this - approximizations can be made for values of x close to a given point

$$f(x) \approx L(x) = f'(a)(x-a) + f(a)$$

Find a linearization of $f(x) = \sqrt{1+x}$ at x = 0

$$f' = \frac{1}{2}(1+x)^{-\frac{1}{2}}\Big|_{x=0} = \frac{1}{2} \qquad (0,1)$$

$$L(x) = \frac{1}{2}(x-0)+1$$

Use the linearization to approximate $\sqrt{1.02}$

$$f(.02) = \sqrt{1+(.02)} = \sqrt{1.02}$$

$$L(.02) = \frac{1}{2}(.02)+1$$

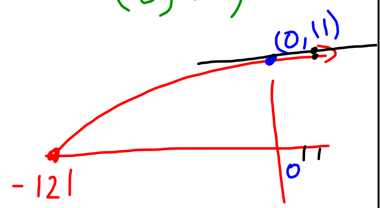
$$= 1.01$$

$$f(.02) \approx 1.01$$

Use linearizations to approximate $\sqrt{123}$

$$(0,11)$$

$$f(x) = \sqrt{121+x}$$



$$f'(x) = \frac{1}{2}(121+x)^{-\frac{1}{2}}$$

$$\frac{1}{2(121+x)^{\frac{1}{2}}} = \frac{1}{2\sqrt{121+x}}\Big|_{x=0} = \frac{1}{22}$$

$$L(x) = \frac{1}{22}(x-0)+11$$

$$L(2) = \frac{1}{22}(2)+11$$

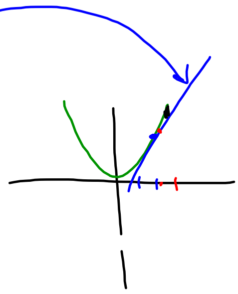
$$\frac{1}{11}+11 = 11\frac{1}{11} = \frac{122}{11}$$

Write a Linearization for $y=x^2$ near $x=2$

$y' = 2x \Big|_{x=2} = 4$

$x = 2.01$

$L(x) = 4(x-2) + 4$



$L(2.01) = 4(2.01-2) + 4$

$= 4(.01) + 4$

approx. 4.04

exact 4.0401

approx $x = 2.5$ $y = 6$ $\longleftarrow$ 6.25

Differentials

$dy = f'(x)dx$ approx. change

$f'(x) = \frac{dy}{dx}$

the differential approximates Δy which is the actual change in y

Δx & Δy - actual change

$2\pi(4)(1)$

Given $A = \pi r^2$ find the differential dA and evaluate dA for r = 10 and dr = .1

$\frac{dA}{\cancel{dr}} = (2\pi r)\,dr$

$dA = 2\pi r\,dr$

$dA = 2\pi(10)\,.1$

$dA = 2\pi$

What does the differential dA represent?

change in area

Volume of a Cube

$V = s^3$

$s = 3 \text{ in}$
$ds = .5 \text{ in}$

$dV = 3s^2\, ds$

$dV = 3(3_{\text{in}})^2(.5_{\text{in}}) = 13.5 \text{ in}^3$

$s = 3$
$ds = -.5 \text{ in}$

$dV = 3(3)^2(-.5)$

$dV = -13.5 \text{ in}^3$

Percent change is represented by $\frac{df}{f(a)} \bullet 100$

$\left(\frac{9950.05}{4\pi(3959)^2}\right)100 = .005\%$

If the radius of the earth is estimated to be $3959 \pm .1$ miles what effect would the tolerance of 0.1 have on an estimate of the earth's surface area? error

$SA = 4\pi r^2$

$dSA = 8\pi r\, dr$

$dSA = 8\pi(3959)(.1)$

$dSA = 9950.05 \text{ mi}^2$