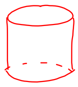
16.



$SA = \pi r^2 + 2\pi r h$

$V = 1000\,\text{cm}^3 = \pi r^2 h$

$r = \frac{10}{\sqrt[3]{\pi}} \longrightarrow h = \frac{1000}{\pi r^2}$ $\quad \frac{1000}{\pi\left(\frac{10}{\pi^{1/3}}\right)^2}$ $\quad \frac{1000}{\pi \frac{100}{\pi^{2/3}}}$ $\quad \frac{10}{\pi^{1/3}}$

$SA = \pi r^2 + 2\pi r\left(\frac{1000}{\pi r^2}\right)$

$SA = \pi r^2 + \frac{2000}{r}$

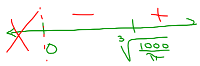
$SA' = \frac{2\pi r \cdot r^2}{r^2} - \frac{2000}{r^2}$

$\frac{2\pi r^3 - 2000}{r^2}$

$2\pi r^3 - 2000 = 0$ $\qquad r^2 = 0$

$\cancel{2\pi} r^3 = \frac{2000}{2\pi}$ $\qquad r = 0$

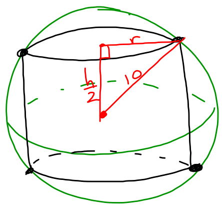
$r = \sqrt[3]{\frac{1000}{\pi}}$



min. @ $r = \sqrt[3]{\frac{1000}{\pi}}$ because

SA' changed from (−) to (+)

22.



$V = \pi r^2 h$

$\left(\frac{h}{2}\right)^2 + r^2 = 100$

$r^2 = 100 - \frac{h^2}{4}$

$h = \frac{20}{\sqrt{3}}$

$V = \pi\left(100 - \frac{h^2}{4}\right)h$

$V = \pi\left(100 - \frac{400}{3 \cdot 4}\right)\frac{20}{\sqrt{3}}$

$\pi\left(100 - \frac{100}{3}\right)\cdot\frac{20}{\sqrt{3}}$ $\quad V = 100\pi h - \frac{\pi h^3}{4}$

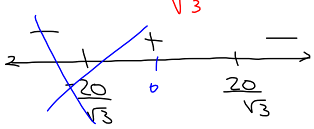
$\pi\,\frac{200}{3}\cdot\frac{20}{\sqrt{3}}$ $\quad V' = 100\pi - \frac{3\pi}{4}h^2 = 0$

$\frac{4000\pi}{3\sqrt{3}}$

$\frac{4}{3}\;\frac{100\cancel{\pi}}{\cancel{\pi}} = \cancel{\frac{3\pi}{4}} h^2$

$\frac{400}{3} = h^2$

$h = \pm\frac{20}{\sqrt{3}}$



59.

max @ $\frac{\cancel{2}}{3}\left(\frac{1}{\cancel{2}}\right)$

$r = \frac{1}{3}$

$V = c(r_0 - r)r^2$ $r_0 = .5$ $c = 1$

$V = (.5 - r)r^2$

$V = cr_0 r^2 - cr^3$

$V' = 2cr_0 r - 3cr^2 = 0$

$cr(2r_0 - 3r) = 0$

$cr = 0$ $2r_0 - 3r = 0$

$r = 0$ $\frac{2r_0}{3} = \frac{\cancel{3}r}{\cancel{3}}$

$-$ $+$ $-$

0 $\frac{2}{3}r_0$

V' has a max @ $r = \frac{2}{3}r_0$

b/c V' changes from

(+) to (−)

4.5 Linearizations, Newton's Method and differentials

Newton's Method

locally linear -

$$y - f(x_1) = f'(x_1)(x - x_1)$$

if you have an x-intercept then

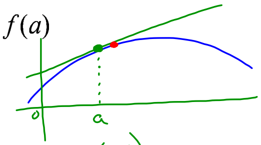
solve for x

Linearizations:

because of this - approximizations can be made for values of x close to a given point

$$f(x) \approx L(x) = f'(a)(x-a) + f(a)$$

$\frac{1}{2}$



$(1+x)^{\frac{1}{2}}$

Find a linearization of $f(x) = \sqrt{1+x}$ at x = 0 $(0,1)$

$f' = \frac{1}{2}(1+x)^{-\frac{1}{2}} \Big|_{x=0} = \frac{1}{2}(1+0)^{-\frac{1}{2}} = \frac{1}{2}$

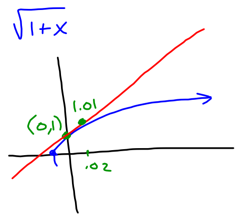
$L(x) = \frac{1}{2}\underline{(x-0)} + \underline{1}$

Use the linearization to approximate $\sqrt{1.02}$

$f(.02) = \sqrt{1+(.02)} = \sqrt{1.02}$

$L(.02) = \frac{1}{2}(.02-0)+1$

$= 1.01$

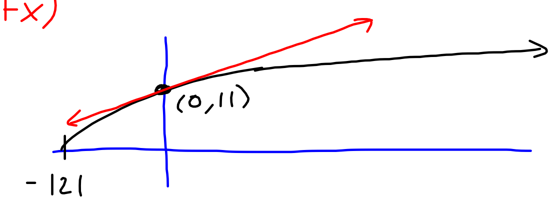


Use linearizations to approximate $\sqrt{123}$

$f(x) = \sqrt{121+x}$

$\frac{1}{2}(121+x)^{-\frac{1}{2}}$

$\frac{1}{2\sqrt{121}}$



$y = \frac{1}{22}(x-0) + 11$

$y = \frac{1}{22}(2) + 11$

$= 11\frac{1}{11}$

Differentials

$dy = f'(x)dx$

the differential approximates Δy which is the actual change in y

Δx & Δy

Given $A = \pi r^2$ find the differential dA and evaluate dA for r = 10 and dr = .1

What does the differential dA represent?

Percent change is represented by $\frac{df}{f(a)} \bullet 100$

If the radius of the earth is estimated to be $3959 \pm .1$ what effect would the tolerance of 0.1 have on an estimate of the earth's surface area?