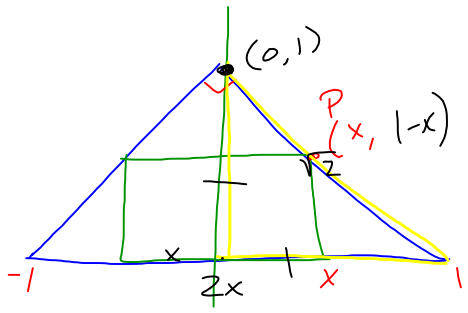
5.



$y = 1 - x$

$1 \times \frac{1}{2}$

$A = 2x(1-x) = 1\left(\frac{1}{2}\right) = \frac{1}{2}$

$A = 2x - 2x^2$

$A' = 2 - 4x$

$x = \frac{1}{2}$

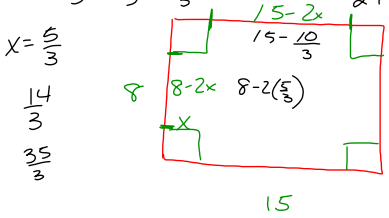
7.

$\frac{5}{3}$ in $\times$ $\frac{14}{3}$ in $\times$ $\frac{35}{3}$ in $\quad V = \frac{2450}{27}$ in^3

$x = \frac{5}{3}$

$\frac{14}{3}$

$\frac{35}{3}$



$V = x(8-2x)(15-2x)$

$V = x(120 - 16x - 30x + 4x^2)$

$V = 120x - 46x^2 + 4x^3$

$V' = 120 - 92x + 12x^2 = 0$

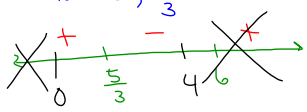
$12x^2 - 92x + 120 = 0$

$4(3x^2 - 23x + 30) = 0$

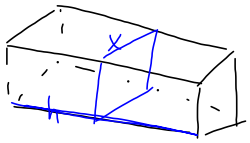
18
5

$(3x - 5)(x - 6) = 0$

$x = 6, \frac{5}{3}$



30.



$18_{in} \times 18_{in} \times 36_{in}$

108

$4x + h = 108$

$h = 108 - 4x$

$V = x^2 h$

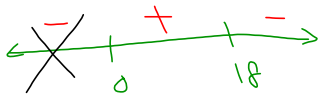
$V = x^2(108 - 4x)$

$V = 108x^2 - 4x^3$

$V' = 216x - 12x^2 = 0$

$12x(18 - x) = 0$

$x = 0, 18$



4.4b Modeling and Optimization

Designing a Can

You have been asked to design a one-liter oil can shaped as a right circular cylinder. What dimensions will use the least material?

$SA = 2\pi r^2 + 2\pi r h$

$V = 1 \text{ liter} = \pi r^2 h$

$1000 \text{ cm}^3 = \pi r^2 h$

$h = \frac{1000}{\pi r^2}$

$r = 5.419$

$h = 10.840$

$SA = 2\pi r^2 + 2\pi r\left(\frac{1000}{\pi r^2}\right)$

$SA = 2\pi r^2 + \frac{2000}{r}$

$SA' = \frac{4\pi r r^2}{r^2} - \frac{2000}{r^2}$

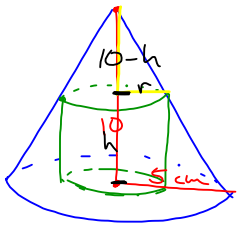
$SA' = \frac{4\pi r^3 - 2000}{r^2} = 0$

$4\pi r^3 - 2000 = 0$ $\quad r^2 = 0$

$\pi r^3 - 500 = 0$ $\quad r = 0$

$r = \sqrt[3]{\frac{500}{\pi}}$

A cylinder is inscribed in a cone of height 10 cm and radius of 5 cm. Find the volume of the largest cylinder.



$V = \pi r^2 h$

$\frac{10-h}{10} = \frac{r}{5}$

$10r = 5(10-h)$

$\frac{10r-50}{-5} = -h$

$-2r+10 = h$

$V = \pi r^2(-2r+10)$

$V = -2\pi r^3 + 10\pi r^2$

$V' = -6\pi r^2 + 20\pi r$

$-6\pi r^2 + 20\pi r = 0$

$-2\pi r(3r-10) = 0$

$-2\pi r = 0 \quad 3r-10=0$

$r = 0, \frac{10}{3}$