9.

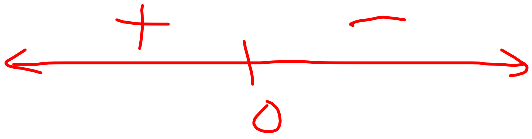
$$y = 2x^{\frac{1}{5}} + 3$$

$$y' = \frac{2}{5}x^{-\frac{4}{5}}$$

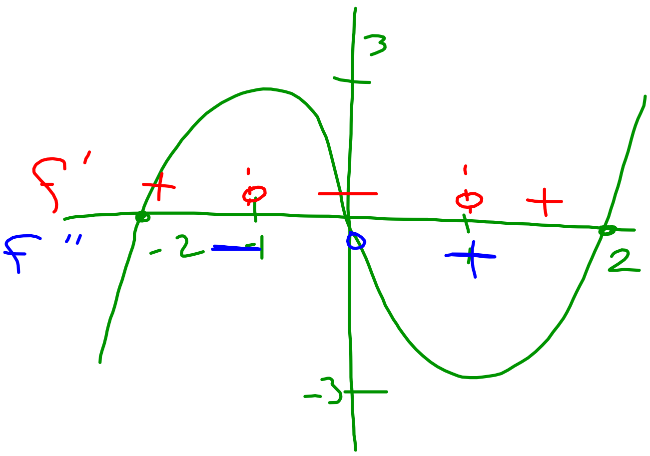
$$y'' = -\frac{8}{25}x^{-\frac{9}{5}} = -\frac{8}{25x^{\frac{9}{5}}}$$

$$25x^{\frac{9}{5}} = 0$$

$$x = 0$$



21.



15.

$$y = \tan^{-1} x$$

$$y' = \frac{1}{1+x^2}$$

$$y'' = \frac{\cancel{(1+x^2)0} - 1(2x)}{(1+x^2)^2}$$

$$y'' = \frac{-2x}{(1+x^2)^2}$$

$$-2x = 0 \qquad (1+x^2)^2 = 0$$

$$x = 0 \qquad 1+x^2 = 0$$

$$x^2 = -1$$

+ +

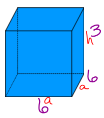
− −

+ −

0

4.4 Optimization

A manufacturer wants to design an open box having a square base and a surface area of 108 sq. in. What dimensions will produce a box with maximum volume?



6in x 6in x 3in

$a = 6$
$h = 3$

$$V = BA \cdot h$$

$$V = a^2 \cdot h$$

$$SA = 4ah + a^2 = 108$$

$$\cancel{4a}h = \frac{108 - a^2}{4a}$$

$$h = \frac{108 - a^2}{4a}$$

$$V = a^2\left(\frac{108 - a^2}{4a}\right)$$

$$V = \frac{108a^2 - a^4}{4a}$$

$$V = \frac{108a - a^3}{4}$$

$$V = \frac{108a}{4} - \frac{a^3}{4}$$

$$V = 27a - \frac{a^3}{4} \qquad \frac{1}{4}a^3$$

$$V' = 27 - \frac{3}{4}a^2 = 0$$

$$\frac{4}{\cancel{3}} \cdot \cancel{27}^{\,9} = \cancel{\frac{3}{4}} a^2$$

$$a^2 = 36$$

$$a = \pm 6$$

− + −

−6 min. 0 6 max

Understand the problem: Identify all given quantities and quantities to be determined. If possible, make a sketch.

Write a Primary Equation to model the problem - represents the quantity to maximize or minimize.

Determine the feasible domain of the primary equation. Graphing is usually helpful!

Reduce the primary equation to 1 independent variable. (may involve the use of secondary equations)

Determine the desired max or min using calculus.

Find the critical points and endpts.

Use the 1st or 2nd derivative test to identify max or min points.

Answer the original question.

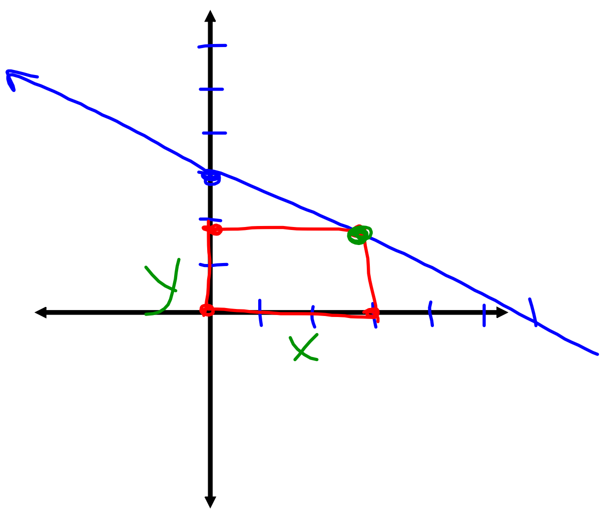
A rectangle is bounded by x & y axes and the graph of $y = \frac{6-x}{2}$

What are the dimensions that maximize the area of the rectangle?

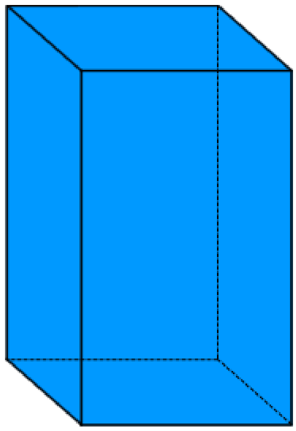
$A = x\,y$

$A = x\left(\frac{6-x}{2}\right)$

$A = 3x - \frac{x^2}{2}$

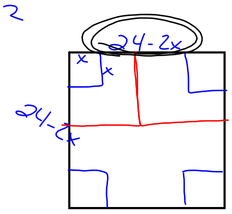


Determine the dimensions of a rectangular solid with a square base and surface area of 337.5 sq. cm. that maximize the volume.



An open top box is to be created by cutting squares from the corners of a 24" square piece of paper and bending up the sides. Determine the dimensions of the box with a maximum volume.

$$V = x(24-2x)^2$$



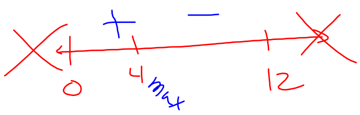
$$V' = x(2(24-2x)(-2)) + (24-2x)^2$$

$$-4x(24-2x) + (24-2x)^2$$

$$(24-2x)(-4x + 24-2x) = 0$$

$$(24-2x)(24-6x) = 0$$

$$x = 12, 4$$



4in × 16in × 16in.

What is the largest rectangular garden that can be enclosed with 600 feet of fence?