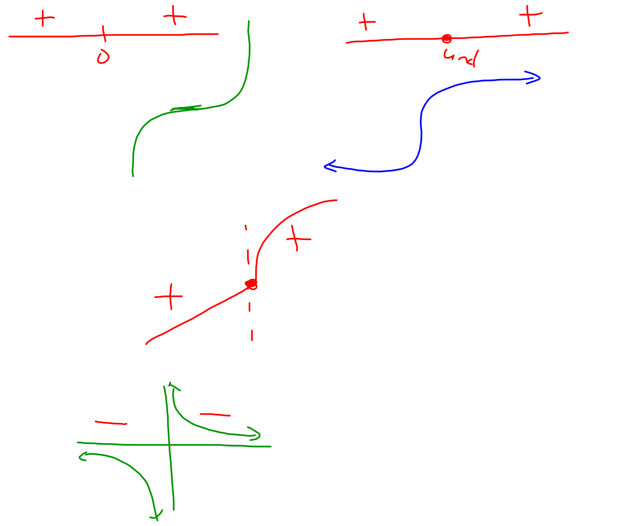
4.3 Connecting f ' & f " with the graph of f

first derivative test for local extrema of continuous functions

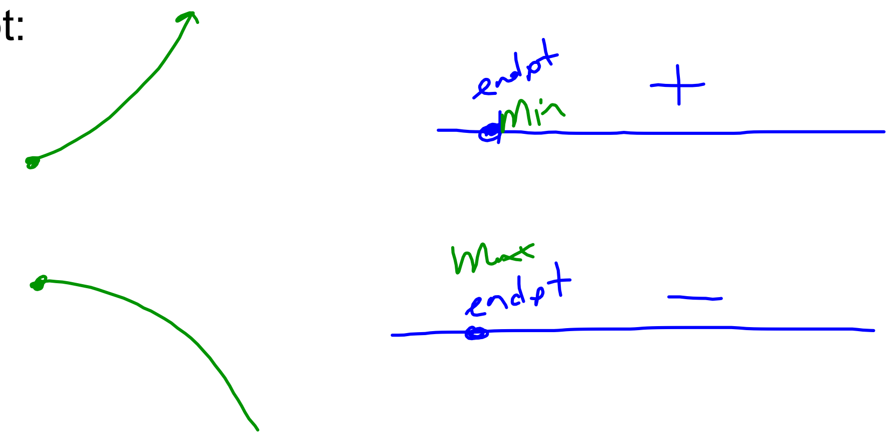
if f' changes from (+) to (-) then f has a max

if f' changes from (-) to (+) then f has a min.

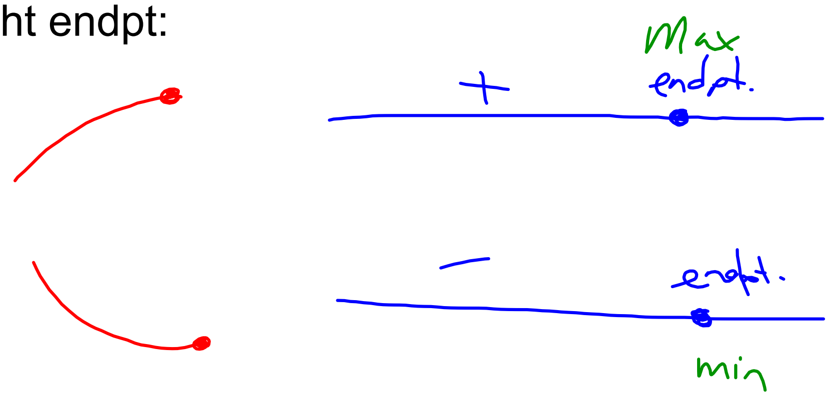
no sign change: flat, vertical tangent, corner, VA cusp



left endpt:



right endpt:



Find the local extrema, increasing, decreasing behavior

$$y = x^3 - 12x - 5$$

$$y' = 3x^2 - 12 = 0$$

$$3x^2 = 12$$

$$x^2 = 4$$

$$x = \pm 2$$

f' + − +

max −2, min 2

11 −21

max of 11 @ $x = -2$

min of −21 @ $x = 2$

inc: $(-\infty, -2) \cup (2, \infty)$

dec: $(-2, 2)$

Find the local extrema $y = (x^2 - 3)e^x$

$$y' = (x^2 - 3)e^x + 2xe^x = 0$$

$$e^x(x^2 - 3 + 2x) = 0$$

$$e^x(x^2 + 2x - 3) = 0$$

$$e^x(x+3)(x-1) = 0$$

$e^x = 0$ $x + 3 = 0$ $x - 1 = 0$

no x's $x = 1, -3$

+ − +

Max −3, min 1

Concavity test

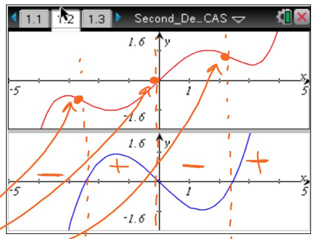
$f'' > 0$ $+$

$f'' < 0$ $-$

f'' changes sign:

f

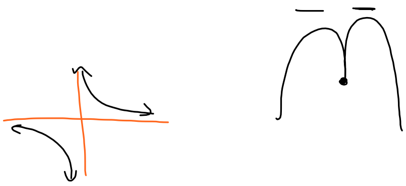
f''



inflection pts.

change in concavity →

$f''=0$, und. and changes sign



Find all points of inflection for:

$$y = e^{-x^2}$$

$$y' = -2xe^{-x^2}$$

$$y'' = -2x\left(-2xe^{-x^2}\right) - 2e^{-x^2}$$

$$= 4x^2e^{-x^2} - 2e^{-x^2} = 0$$

$$2e^{-x^2}(2x^2 - 1) = 0$$

$$2x^2 = 1$$

$$e^{-x^2} = 0 \qquad x = \pm\sqrt{\tfrac{1}{2}}$$

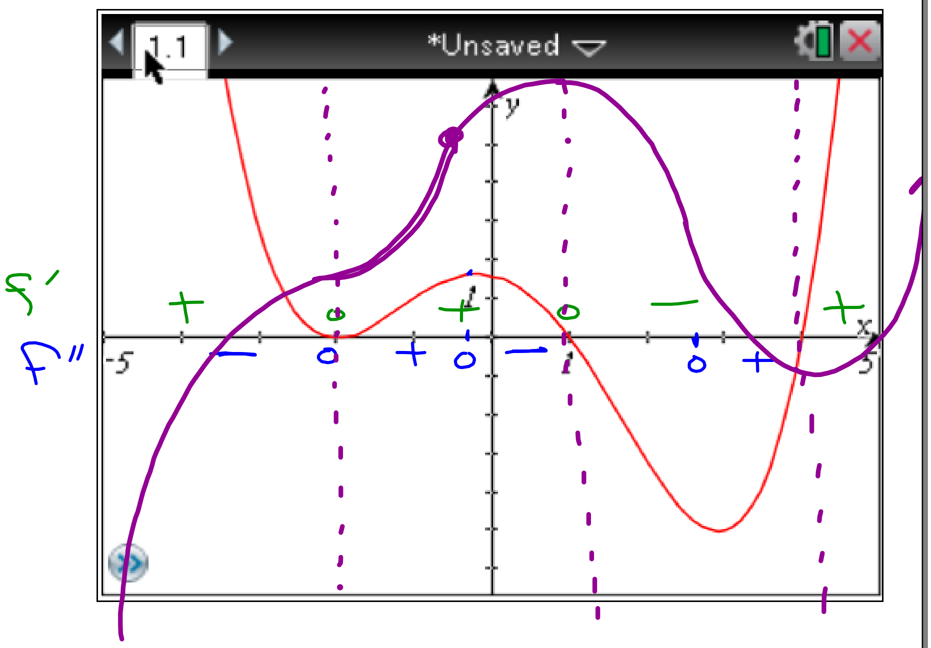
$$\ln 0 = -x^2$$

no x's

$+$ $-$ $+$

$-\frac{1}{\sqrt{2}}$ $\frac{1}{\sqrt{2}}$

Given the graph of f' Sketch a possible graph of *f*.



Second derivative test for local extrema:

if $f' = 0$ & $f'' > 0$ then f has a min.

if $f' = 0$ & $f'' < 0$ then f has a max.

Find the local extrema using the second derivative test:

$$y = \frac{1}{3}x^3 - x$$

$y' = x^2 - 1 = 0$

$x = \pm 1$

f'

f''

Max Min

−1 1

+

$y'' = 2x$

Given the graph of f' Sketch a possible graph of *f*.