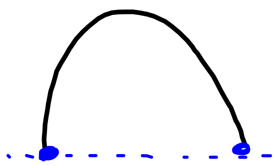
Max when f' changes from
(+) to (−)

Min when f' changes from
(−) to (+)

4.2 Mean Value Theorem (MVT)

Rolle's Theorem:



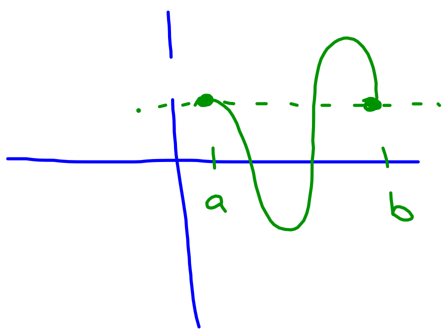
if f(a) = f(b)

cont.

differentiable

then $f'(x) = 0$

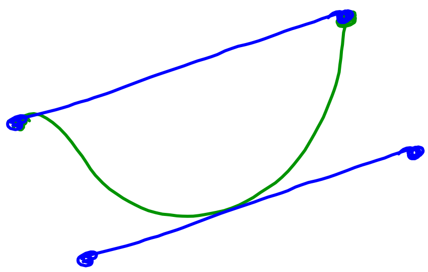
at some point between a & b



On your paper, start at a closed endpt (a) and draw a continuous differentiable function ending at a second closed endpt (b).

Draw the secant line representing the average rate of change.

Can you draw a tangent line to your curve - parallel to the secant line?



MVT:

if f is continuous on a closed interval [a, b] & differentiable on (a, b), then there exists a value "c" in between a & b such that:

instantaneous rate of change

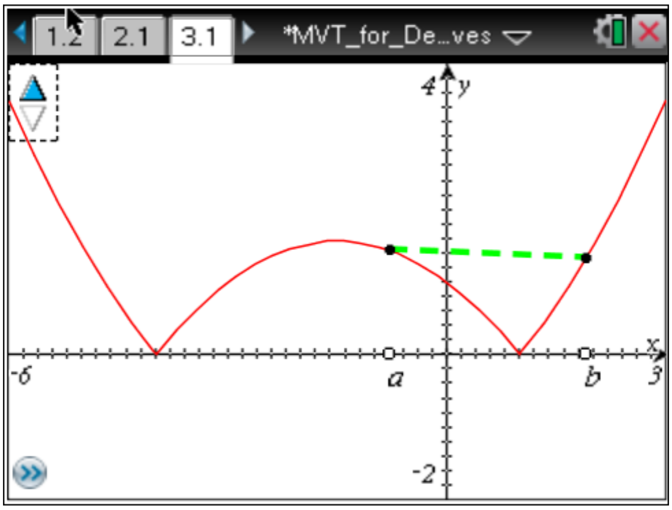
$$f'(c) = \boxed{\frac{f(b) - f(a)}{b - a}}$$

avg. rate of change

slope of secant line

slope of the tangent line

meaning: there is some point where the instantaneous rate of change = the average rate of change



Brian got a speeding ticket driving to the Kansas State football game on Saturday. He left at 1:00 p.m. and was issued a speeding ticket at 1:40 p.m. after driving for 45 miles. Brian claims he never drove over the 65 mile per hour speed limit. Use the Mean Value Theorem to plead Brian's case, or to argue against him.

$$\frac{45\text{ miles}}{2\,\cancel{40}\,\cancel{\text{min}}} \cdot \frac{\overset{3}{\cancel{60}}\,\cancel{\text{min}}}{1\text{ hr.}}$$

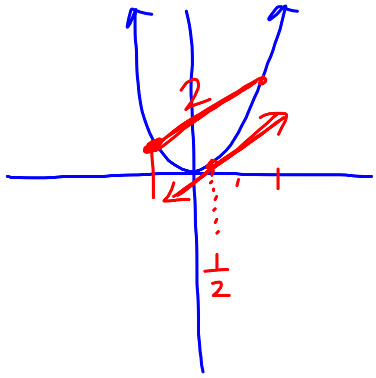
67.5 mph

Show $f(x)=2x^2$ satisfies the MVT over [-1,2] & find a solution

a b

to $f'(c)=\dfrac{f(b)-f(a)}{b-a}$

$$4x = \frac{8-2}{2-(-1)}$$



$$4x = 2$$

$$x = \frac{1}{2}$$

$f(x)=\sqrt{1-x^2}$ on [-1,1]

a b

$$\frac{1}{\cancel{2}}(1-x^2)^{-\frac{1}{2}}(-\cancel{2}x) = \frac{0-0}{1-(-1)}$$

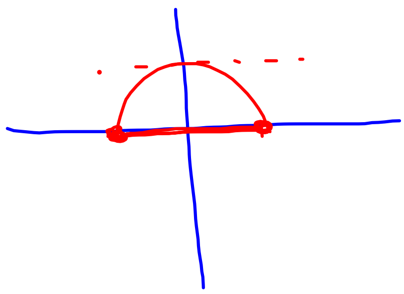
$$x^2+y^2=1$$

$$y^2=1-x^2$$

$$y=\pm\sqrt{1-x^2}$$

$$\frac{-x}{\sqrt{1-x^2}} = 0$$

$$x=0$$

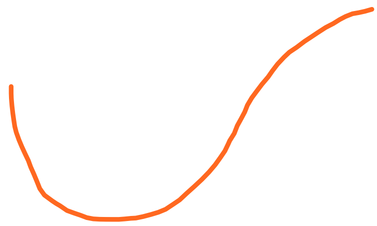


Increasing & decreasing

$f'(x) > 0$ increasing f

$f'(x) < 0$ decreasing f

$f'(x) = 0$ max, min, flat spot



Where is $f(x) = x^3 - 4x$ increasing & decreasing?

$f'(x) = 3x^2 - 4 = 0$

$x^2 = \frac{4}{3}$

$x = \pm\sqrt{\frac{4}{3}}$

+ − +

max min

$-\sqrt{\frac{4}{3}}$ $\sqrt{\frac{4}{3}}$

$f(x) = \ln(x-1)$ $[2,4]$

$\frac{1}{x-1} \cdot 1 = \frac{\ln 3 - 0}{4-2}$

$\frac{1}{x-1} = \frac{\ln 3}{2}$

$x - \cancel{1} = \frac{2}{\ln 3} + 1$