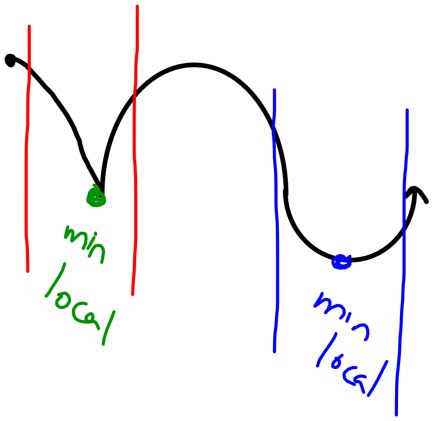
4.1 Extreme Values of Functions

Extrema

local (relative)

global (absolute)



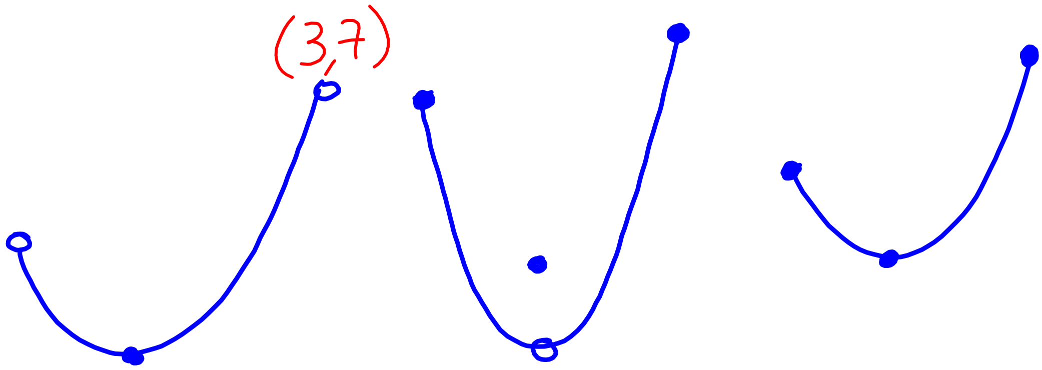
Continuous

Function

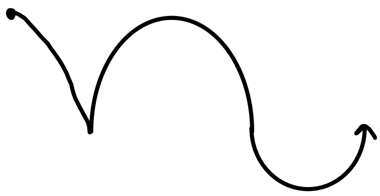
closed interval

Extreme Value Theorem (EVT)

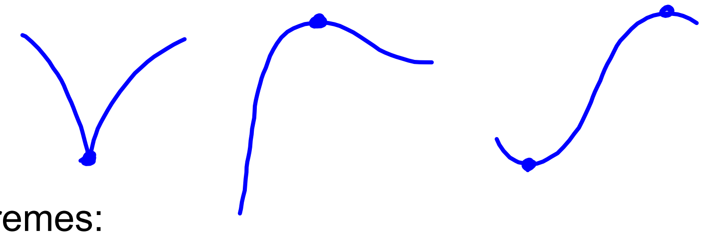
if *f* is continuous on the closed interval [a, b], then *f* has both a max and a min on the interval.



Candidates for max/min?



Critical points
endpoints



Candidates for extremes:

1. $f'(x) = 0$
2. $f'(x) =$ und.
3. endpts.

Ex. Find the extrema for $f(x) = 3x^4 - 4x^3$ on [-1, 2]

$$f'(x) = 12x^3 - 12x^2$$

1. $f'(x) = 0$

$$12x^3 - 12x^2 = 0$$
$$12x^2(x-1) = 0$$
$$12x^2 = 0 \qquad x - 1 = 0$$
$$x = 0, 1$$

2. $f'(x) = \text{und.}$

3. $x = -1, 2$

endpts: $-1, 2$
critical pts: $0, 1$

	−		−		+	
x	-1		0		1	2
	$3(-1)^4 - 4(-1)^3$					
$f(x)$	7		0		-1	16
					min	max

Ex. Find the extrema for: $f(x) = 2x - 3x^{\frac{2}{3}}$ on [-1, 3]

$$f' = 2 - 2x^{-\frac{1}{3}}$$

1. $2 - 2x^{-\frac{1}{3}} = 0$

$$2\left(1 - x^{-\frac{1}{3}}\right) = 0$$

$$2 - \frac{2}{\sqrt[3]{x}} = 0$$
$$2\left(1 - \frac{1}{\sqrt[3]{x}}\right) = 0$$
$$\frac{\sqrt[3]{x} - 1}{\sqrt[3]{x}} = 0$$
$$\sqrt[3]{x} - 1 = 0$$
$$\sqrt[3]{x} = 1$$
$$x = 1$$

$$1 - \frac{1}{\sqrt[3]{x}} = 0 \leftarrow 1 - x^{-\frac{1}{3}} = 0$$
$$1 = x^{-\frac{1}{3}}$$
$$(1)^3 = \left(\frac{1}{x^{\frac{1}{3}}}\right)^3$$
$$1 = \frac{1}{x}$$
$$x = 1$$

2. $\sqrt[3]{x} = 0$

$$x = 0$$

$$f(x) = 2x - 3x^{\frac{2}{3}}$$

x	-1	0	1	3
	$2(-1) - 3(1)$		$2(1) - 3(1)$	$2(3) - 3(3)^{\frac{2}{3}}$
$f(x)$	-5	0	-1	$0 -$
	min	max		

$$f(x)=\frac{1}{\sqrt{4-x^2}}$$

$$f'=\frac{x}{(4-x^2)^{\frac{3}{2}}}$$

1. $x=0$

2. $\left((4-x^2)^{\frac{3}{2}}\right)^{\frac{2}{3}}=0^{\frac{2}{3}}$

$4-x^2=0$

$x=\pm 2$

$$f(x)=\frac{1}{\sqrt{4-x^2}}$$

$-$ $+$

min

-2 0 2

und $\frac{1}{2}$ und.

$$f(x)=\begin{cases}5-2x^2, x\le 1\\ x+2,\ x>1\end{cases}$$