17. $y = \sec^{-1}\left(\frac{1}{t}\right) \qquad 0 < t < 1$

$$y' = \frac{1}{\left|\frac{1}{t}\right|\sqrt{\left(\frac{1}{t}\right)^2 - 1}} \cdot -1t^{-2}$$

$$\frac{-1}{\frac{1}{\cancel{t}} \cdot t^{\cancel{2}}\sqrt{\frac{1}{t^2} - 1}}$$

27.

$$\frac{-1}{t\sqrt{\frac{1}{t^2} - \frac{t^2}{t^2}}} = \frac{-1}{t\sqrt{\frac{1-t^2}{t^2}}}$$

$$\frac{-1}{\cancel{t}\,\frac{\sqrt{1-t^2}}{\cancel{t}}} = \frac{-1}{\sqrt{1-t^2}}$$

21. $y = \tan^{-1}\sqrt{x^2-1} + \csc^{-1} x \qquad x > 1$

$$y' = \frac{1}{1 + (\cancel{\sqrt{x^2-1}})^{\cancel{2}}} \cdot \frac{1}{\cancel{2}}(x^2-1)^{-\frac{1}{2}} \cdot \cancel{2}x \quad - \frac{1}{|x|\sqrt{x^2-1}}$$

$$\frac{1}{x^{\cancel{2}}} \cdot \frac{\cancel{x}}{\sqrt{x^2-1}} - \frac{1}{x\sqrt{x^2-1}}$$

$$\frac{1}{x\sqrt{x^2-1}} - \frac{1}{x\sqrt{x^2-1}} = 0$$

23. $y = \sec^{-1} x$

$y' = \frac{1}{|x|\sqrt{x^2-1}}\Big|_{x=2} = \frac{1}{2\sqrt{3}}$

$y = \frac{1}{2\sqrt{3}}(x-2) + \frac{\pi}{3}$

$x = 2$

$\left(2, \frac{\pi}{3}\right)$

$y = \sec^{-1} 2$

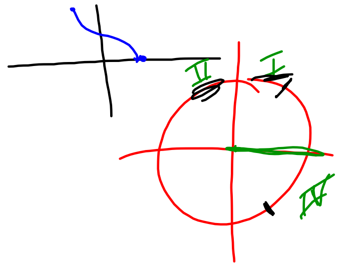
$\sec y = 2$

$\frac{1}{\cos y} = 2$

$\cos y = \frac{1}{2}$

$y = \cos^{-1}\left(\frac{1}{2}\right)$

$\frac{\pi}{3}, \frac{5\pi}{3}$



27a. $y = \tan x$

$\left(\frac{\pi}{4}, 1\right)$

$y' = \sec^2 x \Big|_{x=\frac{\pi}{4}} = 2$

$\frac{1}{\left(\cos\frac{\pi}{4}\right)^2}$

$\frac{\sqrt{2}}{2}$

$y = 2\left(x - \frac{\pi}{4}\right) + 1$

b. $y = \tan^{-1}(x)$

$\left(1, \frac{\pi}{4}\right)$

$\frac{1}{1+x^2}$

$y = \frac{1}{2}(x-1) + \frac{\pi}{4}$

$$y = \tan x$$

$$x = \tan^{-1} y \qquad \left(\frac{\pi}{4}, 1\right)$$

$$1 = \frac{1}{1+y^2} \cdot \frac{dy}{dx}$$

$$1 = \frac{1}{2} \cdot \frac{dy}{dx}$$

$$y = \underline{\underline{\cot}}^{-1}(\sqrt{3}) \qquad y = \csc^{-1}(2)$$

$$\cot y = \sqrt{3} \qquad \sin^{-1}\left(\frac{1}{2}\right)$$

$$\frac{1}{\tan y} = \sqrt{3}$$

$$\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

3.9b Derivatives of logarithms

Properties of Logs:

$y = \log_b x \qquad b^y = x$

$y = \ln_e x$ means $e^y = x$

$y = \log_{10} x \qquad 10^y = x$

$\ln e = 1$

$\log(ab) = \log a + \log b$

$\log\left(\frac{a}{b}\right) = \log a - \log b$

$\log a^n = n \log a$

$\ln e^x = x \ln e = x$

$e^{\ln x} = x$

$2^{\log_2 7} = 7$

$e^{\ln 5} = 5$

Derivative of $y=\ln(x)$

means $e^y = x$

$e^y \cdot \frac{dy}{dx} = 1$

$\frac{dy}{dx} = \frac{1}{e^y} = \frac{1}{x}$

general:

$\frac{d}{dx}(\ln u) = \frac{1}{u} \cdot \frac{du}{dx}$

Find $\frac{dy}{dx}$

1. $y=\ln(2x)$

$y' = \frac{1}{2x} \cdot 2 = \frac{1}{x}$

2. $y=\ln\left(\frac{3}{x}\right)$

$y' = \frac{1}{\frac{3}{x}} \cdot -3x^{-2}$

$\frac{x}{\cancel{3}} \cdot -\cancel{3}x^{-2} = -x^{-1} = \frac{-1}{x}$

Log properties to aid:

$\frac{1}{\sqrt{x+1}} \cdot \frac{1}{2}(x+1)^{-\frac{1}{2}} \cdot 1$

rewrite:

1. $f(x) = \ln\sqrt{x+1}$

$\ln(x+1)^{\frac{1}{2}}$

$\frac{1}{2}\ln(x+1)$

$f'(x) = \frac{1}{2}\left(\frac{1}{x+1} \cdot 1\right)$

$= \frac{1}{2(x+1)}$

2. $f(x) = \ln\frac{x(x^2+1)^2}{\sqrt{2x^3-1}}$

$\ln x + 2\ln(x^2+1) - \frac{1}{2}\ln(2x^3-1)$

$f'(x) = \frac{1}{x} + 2\left(\frac{1}{x^2+1} \cdot 2x\right) - \frac{1}{2}\left(\frac{1}{2x^3-1} \cdot 6x^2\right)$

$\frac{1}{x} + \frac{4x}{x^2+1} - \frac{3x^2}{2x^3-1}$

Change of Base:

$y = \log_b x \Rightarrow \frac{\ln x}{\ln b} = \frac{1}{\ln b} \cdot \ln x$

Derivative of $y = \log_b x$

$\frac{d}{dx}(\log_b x) = \frac{d}{dx}\left(\frac{\ln x}{\ln b}\right) = \frac{d}{dx}\left(\frac{1}{\ln b} \cdot \ln x\right)$

$\frac{1}{\ln b}\left(\frac{1}{x} \cdot 1\right) = \frac{1}{x \ln b}$

general:

$\frac{d}{dx}\left(\log_b u\right) = \frac{1}{u} \cdot \frac{1}{\ln b} \cdot \frac{du}{dx}$

Find $\frac{dy}{dx}$

1. $y = \log_2(\sin(x))$

$$y' = \frac{1}{\sin x} \cdot \frac{1}{\ln 2} \cdot \cos x$$

2. $y = x^3 \log_5(2x+1)$

$$y' = x^3\left(\frac{1}{2x+1} \cdot \frac{1}{\ln 5} \cdot 2\right) + \log_5(2x+1) \cdot 3x^2$$

$$\frac{2x^3}{(2x+1)\ln 5} + 3x^2 \log_5(2x+1)$$

Logarithmic differentiation:

1. write equation
2. take natural log of each side
3. use log properties to rewrite
4. use implicit differentiation
5. simplify and solve for $\frac{dy}{dx}$
6. substitute for y
7. simplify

$$y = x^x$$

$$\ln y = \ln x^x$$

$$\ln y = x \ln x$$

$$\frac{1}{y} \cdot \frac{dy}{dx} = x\frac{1}{x} + \ln x \cdot 1$$

$$y \cdot \frac{1}{\cancel{y}} \frac{dy}{dx} = (1 + \ln x)y$$

$$\frac{dy}{dx} = (1 + \ln x)\, y$$

$$\frac{dy}{dx} = x^x(1 + \ln x)$$

$$\ln y = \ln \frac{\sqrt{2x+1}\,(x+3)^5}{(x-7)^2}$$

$$\ln y = \frac{1}{2}\ln(2x+1) + 5\ln(x+3) - 2\ln(x-7)$$

$$\frac{1}{y}\frac{dy}{dx} = \left(\qquad \qquad \right)$$

$$\frac{\sqrt{2x+1}\,(x+3)^5}{(x-7)^2}\left(\qquad \qquad \right)$$