3.7b Questions

26

45

56

57

Quiz

1-5 wkst

$$\frac{dy}{dx} = \frac{1}{y+1}$$

$$\frac{d^2y}{dx^2} = \frac{\cancel{(y+1)(0)} - 1\left(\frac{dy}{dx}\right)}{(y+1)^2}$$

$$= \frac{-1}{(y+1)^3}$$

#2

$$y - x\sin y = 3$$

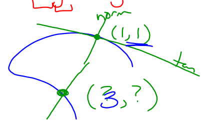
$$\frac{dy}{dx} - \left(x\cos y\frac{dy}{dx} + \sin y\right) = 0$$

$$\frac{dy}{dx} - x\cos y\frac{dy}{dx} - \sin y = 0$$

$$\frac{dy}{dx}\left(\cancel{1 - x\cos y}\right) = \frac{\sin y}{1 - x\cos y}$$

56.

norm $x^2 + 2xy - 3y^2 = 0$ $(1,1)$



$$2x + 2x\frac{dy}{dx} + y\cdot 2 - 6y\frac{dy}{dx} = 0$$

$$\cancel{2}\ 2\frac{dy}{dx} + \cancel{2} - 6\frac{dy}{dx} = 0$$

$$4 - 4\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = 1$$

normal

$$y = -1(x-1)+1$$

$$y = -x+2$$

Substitution

$$x^2 + 2x(-x+2) - 3(-x+2)^2 = 0 \qquad x^2 - 4x + 4$$

$$x^2 - 2x^2 + 4x - 3x^2 + 12x - 12 = 0$$

$$-4x^2 + 16x - 12 = 0$$

$$-4(x^2 - 4x + 3) = 0$$

$$-4(x-1)(x-3) = 0$$

$x = 1, 3 \leftarrow$ substitution into 1 of 2 system original eq.

$(1,1)$ $(3,-1)$

57.

norms $xy + 2x - y = 0$

$\parallel$ $2x + y = 0$

$m = -2$ ← normal line

$m_\perp = \frac{1}{2}$ ← tangent line

$$x\frac{dy}{dx} + y + 2 - \frac{dy}{dx} = 0$$

$$\frac{dy}{dx}\cancel{(x-1)} = \underset{\text{tangent}}{\boxed{\frac{-y-2}{x-1}}}$$

$$\frac{-y-2}{x-1} \cancel{=} \frac{1}{2} \qquad + \left(\frac{x-1}{-y-2}\right) = \frac{+2}{1}$$

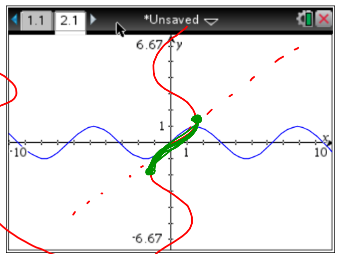
$$1(x-1) = 2(-y-2)$$

$$x-1 = -2y-4$$

3.8 Derivatives of Trig Inverse Functions

Derivative of the Arcsine

$y = \sin^{-1}(x)$ means $x = \sin y$



$$1 = \cos y \cdot \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{1}{\cos y}$$

$$\sin^2 y + \cos^2 y = 1$$

$$\cos y = \pm\sqrt{1-\sin^2 y}$$

$$\cos y = \pm\sqrt{1-x^2}$$

$$\frac{dy}{dx} = \frac{1}{\pm\sqrt{1-x^2}}$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\left(\sin^{-1} u\right) = \frac{1}{\sqrt{1-u^2}} \cdot \frac{du}{dx}$$

$$\frac{d}{dx}\left(\sin^{-1} x^2\right) = \frac{1}{\sqrt{1-(x^2)^2}} \cdot 2x$$

$$\frac{2x}{\sqrt{1-x^4}}$$

$$\frac{1}{3}x^{\frac{1}{2}}$$

$$\frac{d}{dx}\left(\sin^{-1} \frac{\sqrt{x}}{3}\right) = \frac{1}{\sqrt{1-\left(\frac{\sqrt{x}}{3}\right)^2}} \cdot \frac{1}{6}x^{-\frac{1}{2}}$$

$$\frac{1}{6\sqrt{x}\sqrt{1-\frac{x}{9}}} = \frac{1}{6\sqrt{x-\frac{x^2}{9}}}$$

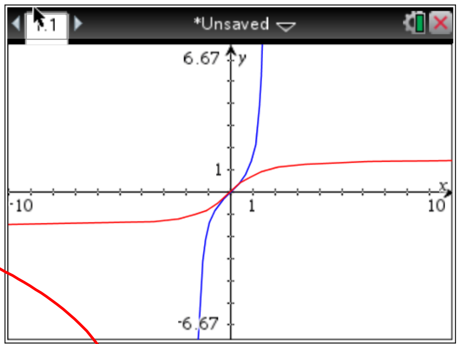
Derivative of the Arctangent

$$y = \tan^{-1} x$$

$$x = \tan y$$

$$1 = \sec^2 y \cdot \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{1}{\sec^2 y}$$



What is the range of $y = \tan^{-1}(x)$?

$$1 + \tan^2 y = \sec^2 y$$

$$1 + x^2 = \sec^2 y$$

$$\frac{dy}{dx} = \frac{1}{1+x^2}$$

A particle moves along the x-axis so that its position at any time $t \geq 0$ is $x(t) = \tan^{-1}\sqrt{t}$. What is the velocity of the particle when t =16?

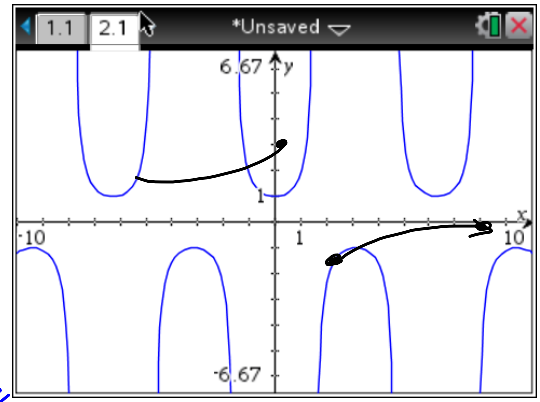
$$\frac{dx}{dt} = \frac{1}{1+(\sqrt{t})^2} \cdot \frac{1}{2} t^{-\frac{1}{2}} \Big|_{t=16}$$

$$= \frac{1}{1+16} \cdot \frac{1}{2} \cdot \frac{1}{4}$$

$$= \frac{1}{17} \cdot \frac{1}{2} \cdot \frac{1}{4} = \frac{1}{136}$$

Derivative of the Arcsecant

$$y = \sec^{-1} x$$



$$x = \sec y$$

$$1 = \sec y \tan y \cdot \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{1}{\sec y \tan y}$$

$$1 + \tan^2 y = \sec^2 y$$

$$\tan y = \pm\sqrt{\sec^2 y - 1}$$

$$\frac{dy}{dx} = \frac{1}{|x|\sqrt{x^2-1}}$$

$$\frac{d}{dx}\left(\sec^{-1}\left(5x^4\right)\right) = \frac{1}{|5x^4|\sqrt{(5x^4)^2-1}} \cdot 20x^3$$

$$\frac{20x^3}{|5x^4|\sqrt{25x^8-1}}$$

Derivatives of the other 3 inverse functions:

$$\cos^{-1}x = \frac{\pi}{2} - \sin^{-1}x$$

$$\frac{-1}{\sqrt{1-x^2}}$$

$$\cot^{-1}x = \frac{\pi}{2} - \tan^{-1}x$$

$$\frac{-1}{1+x^2}$$

$$\csc^{-1}x = \frac{\pi}{2} - \sec^{-1}x$$

$$\frac{-1}{|x|\sqrt{x^2-1}}$$

Derivative of an Inverse Function

If *f* is a function that is differentiable and its inverse is *g*, then *g* is differentiable at any *x* that $f'(g(x)) \neq 0$

$g'(x) = \frac{1}{f'(g(x))}$ meaning $g'(x) = \frac{1}{f'(y)} = \frac{1}{\frac{dx}{dy}}$ i.e. they have reciprocal slopes

Let *f* be the function defined by $f(x) = x^3 + x$.

If $g(x) = f^{-1}(x)$ & $g(2) = 1$, what is the value of $g'(2)$?