7.

$$x + \tan(xy) = 0$$

$$\cancel{x} + \cancel{\sec^2(xy)}\left(x\frac{dy}{dx} + \cancel{y}\right) = 0$$

13.

15.

23.

$$\frac{dy}{dx} = \frac{\frac{-1}{\sec^2(xy)} - y}{x}$$

$$= \frac{-\cos^2(xy) - y}{x}$$

13.

$$x^2y - xy^2 = 4$$

$$x^2\frac{dy}{dx} + y\,2x - \left(x\,2y\frac{dy}{dx} + y^2\right) = 0$$

$$x^2\frac{dy}{dx} - 2xy\frac{dy}{dx} = -2xy + y^2$$

$$\frac{dy}{dx}\left(x^2 - 2xy\right) = \frac{-2xy + y^2}{x^2 - 2xy}$$

$$= \frac{2xy - y^2}{2xy - x^2}$$

$$x^2 - 2xy = 0$$

$$x(x - 2y) = 0$$

$$x = 0 \qquad x - 2y = 0$$

$$y = \frac{x}{2}$$

15.

$$\frac{dy}{dx} = \frac{3x^2 - y}{x - 3y^2}$$

$$x - 3y^2 = 0$$

$$x \neq 3y^2 \qquad y^2 \neq \frac{x}{3}$$

23.

$$\underbrace{2xy} + \pi \sin y = 2\pi \qquad \left(1, \frac{\pi}{2}\right)$$

$$2x\frac{dy}{dx} + y \cdot 2 + \cancel{\pi(\cos y)\left(\frac{dy}{dx}\right)} = 0$$

$$2\frac{dy}{dx} + \pi = 0$$

$$\frac{dy}{dx} = -\frac{\pi}{2}$$

tangent:

$$y = -\frac{\pi}{2}(x-1) + \frac{\pi}{2}$$

normal:

$$y = \frac{2}{\pi}(x-1) + \frac{\pi}{2}$$

3.7b Implicit Differentiation

Show that $\frac{dy}{dx}$ is defined at every point on the graph of

$$2y = x^2 + \sin(y)$$

$$2\frac{dy}{dx} = 2x + (\cos y)\left(\frac{dy}{dx}\right)$$

$$2\frac{dy}{dx} - (\cos y)\frac{dy}{dx} = 2x$$

$$\frac{dy}{dx}\cancel{(2-\cos y)} = \frac{2x}{2-\cos y}$$

$$2 - \cos y = 0$$

$$\cos y = 2 \qquad y = \cos^{-1}(2)$$

no sol.

Graph the curve using parametric equations

$$2y = x^2 + \sin(y)$$

$$x^2 = 2y - \sin y$$

$$x = \pm\sqrt{2y - \sin y}$$

$$x = \sqrt{2y - \sin y} \text{ or } -\sqrt{2y - \sin y}$$

$$x = \sqrt{2t - \sin t} \text{ or } x = -\sqrt{2t - \sin t}$$

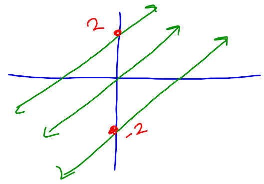
$$y = t \qquad y = t$$

$$x^2 - 2xy + y^2 = 4$$

$$2x - \left(2x\frac{dy}{dx} + 2y\right) + 2y\frac{dy}{dx} = 0$$

find $\frac{dy}{dx}$ $\qquad \frac{dy}{dx} = \frac{2y-2x}{2y-2x} = 1$

Use $\frac{dy}{dx}$ to sketch a possible graph of the implicit curve



Factor the left side and solve for y.
How does this compare with your graph?

$$x^2 - 2xy + y^2 = 4$$

$$(x - y)(x - y)$$

$$\sqrt{(x-y)^2} = \sqrt{4}$$

$$x - y = \pm 2$$

$$x = \pm 2 + y$$

$$y = x \pm 2$$

Find the slope of the Folium of Descartes at the points (4,2) and (2,4).

$$x^3 + y^3 - 9xy = 0$$

$$3x^2 + 3y^2\frac{dy}{dx} - 9x\frac{dy}{dx} - 9y = 0$$

$$\frac{dy}{dx} = \frac{9y - 3x^2}{3y^2 - 9x} = \frac{3y - x^2}{y^2 - 3x}\Bigg|_{(4,2)}^{(2,4)}$$

$$m = \frac{6-16}{4-12} = \frac{-10}{-8} = \frac{5}{4} \qquad m = \frac{12-4}{16-6} = \frac{8}{10} = \frac{4}{5}$$

Find the points where the folium has:

a) a horizontal tangent

b) a vertical tangent

$$\frac{dy}{dx} = \frac{3y - x^2}{y^2 - 3x} = 0$$

$$y^2 - 3x = 0 \qquad 3y - x^2 = 0$$

$$y^2 = 3x \qquad 3y = x^2$$

Folium of Descartes