32. $y = 9\tan\left(\frac{x}{3}\right)$

$y' = 9\sec^2\left(\frac{x}{3}\right)\cdot\frac{1}{3}$

$y' = 3\left(\sec\left(\frac{x}{3}\right)\right)^2$

$y'' = \cancel{3}\left(2\sec\left(\frac{x}{3}\right)\right)\left(\sec\left(\frac{x}{3}\right)\tan\left(\frac{x}{3}\right)\right)\cdot\frac{1}{\cancel{3}}$

$= 2\sec^2\left(\frac{x}{3}\right)\tan\left(\frac{x}{3}\right)$

49. $y = \sin t$

$x = t^2 + t$

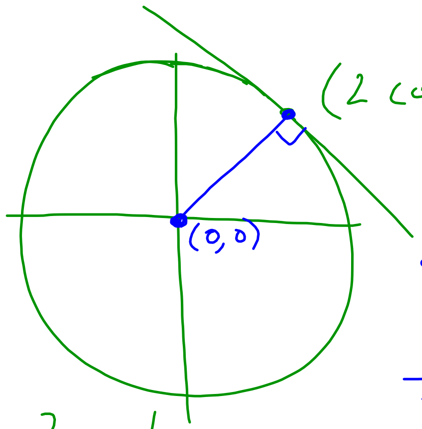
a. $\dfrac{\frac{dy}{dt}}{\frac{dx}{dt}} = \dfrac{dy}{dx} = \dfrac{\cos t}{2t+1}$

b. $\dfrac{d}{dt}\left(\dfrac{dy}{dx}\right) = \dfrac{(2t+1)(-\sin t) - (\cos t)(2)}{(2t+1)^2}$

c. $\dfrac{d}{dx}\left(\dfrac{dy}{dx}\right)$ $\dfrac{\frac{d^2y}{dt\,dx}}{\frac{dx}{dt}}$ $\dfrac{d^2y}{\cancel{dt}\,dx}\cdot\dfrac{\cancel{dt}}{dx}$

$\dfrac{\frac{d^2y}{\cancel{dt}\,dx}\cdot\frac{\cancel{dt}}{dx}}{\cancel{\frac{dx}{dt}}} = \boxed{\dfrac{\frac{(2t+1)(-\sin t) - (\cos t)(2)}{(2t+1)^2}}{2t+1}}$

50.



Slope of Radius

$$\frac{\cancel{2}\sin t}{\cancel{2}\cos t}$$

$x = 2\cos t$

$y = 2\sin t$

Slope of tangent line:

$$\frac{\cancel{2}\cos t}{-\cancel{2}\sin t} = -\frac{\cos t}{\sin t}$$

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$x = 2$

$f(g(x))$

$f'(g(x)) \cdot g'(x)$

$f'(2) \quad -3$

$\frac{1}{3} \cdot -3 = -1$

$$y = \underbrace{37 \sin\left(\frac{2\pi}{365}(x-101)\right)}\ \underbrace{+25}$$

$$y' = 37\left(\cos\left(\frac{2\pi}{365}(x-101)\right)\right)\cdot\frac{2\pi}{365}$$

3.7 Implicit Differentiation

Implicit vs. Explicit

$\frac{dy}{dx}$

$$y = -x^2 + x \qquad\qquad y + x^2 - x = 0$$

$$1\cdot\frac{dy}{dx} = -2x+1 \qquad\qquad 1\,\frac{dy}{dx} + 2x - 1 = 0$$

$$\frac{dy}{dx} = -2x+1$$

Find $\frac{dy}{dx}$ $x^2 + y^2 = 25$

$$y = -x^2 + x$$

$$1 \cdot \frac{dy}{dx} = -2x + 1$$

$\frac{dy}{dx}$

$$y + x^2 - x = 0$$

$$1 \cdot \frac{dy}{dx} + 2x - 1 = 0$$

$$\frac{dy}{dx} = -2x + 1$$

Process for Implicit Differentiation

1. Differentiate both sides of the equation with respect to *x*.

2. Collect the terms with $\frac{dy}{dx}$ on one side of the equation

3. Factor out $\frac{dy}{dx}$

4. Solve for $\frac{dy}{dx}$

Find $\frac{dy}{dx}$ $$y^3 + y^2 - 5y - x^2 = -4$$

$$3y^2\frac{dy}{dx} + 2y\frac{dy}{dx} - 5\cancel{y}\frac{dy}{dx} - 2x = 0$$

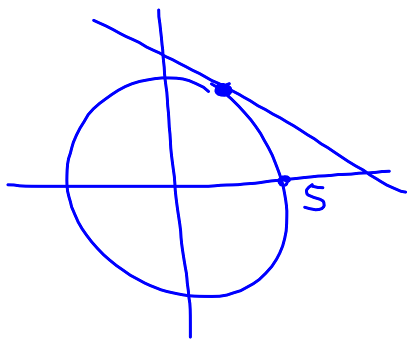
$$\frac{dy}{dx}\left(3y^2 + 2y - 5\right) = 2x$$

$$\frac{dy}{dx} = \frac{2x}{3y^2 + 2y - 5}$$

Find the slope of the circle at the point (3,4)

$$x^2 + y^2 = 25$$

$$\frac{dy}{dx} = \frac{-x}{y}\Bigg|_{(3,4)} = \frac{-3}{4}$$



higher order derivatives

Find the second derivative of y with respect to x

$$x^2 + y^2 = 25$$

$$\frac{dy}{dx} = \frac{-x}{y}$$

$$\frac{d^2y}{dx^2} = \frac{y(-1) - (-x)\frac{dy}{dx}}{y^2}$$

$$= \frac{-y + x\frac{dy}{dx}}{y^2}$$

$$= \frac{\left(-y + x\left(\frac{-x}{y}\right)\right)}{y^2} \frac{1}{y^2}$$

$$\frac{-y}{y^2} - \frac{x^2}{y^3}$$

$$2x^3 - 3y^2 = 8$$

$$6x^2 - 6y\,y' = 0$$

$$y' = \frac{-6x^2}{-6y} = \frac{x^2}{y}$$

$$xy^2 + x^2y - 4x = 10$$

$$x\,2y\frac{dy}{dx} + y^2(1) + x^2\frac{dy}{dx} + y(2x) - 4 = 0$$

$$2xy\frac{dy}{dx} + x^2\frac{dy}{dx} = 4 - 2xy - y^2$$

$$\frac{dy}{dx}\left(\cancel{2xy + x^2}\right) = \frac{4 - 2xy - y^2}{2xy + x^2}$$