Warm-Up

1997MC AB12

At what point on the graph of $y = \frac{1}{2}x^2$ is the tangent line parallel to the line $2x - 4y = 3$?

(a) (0.5, -0.5) (b) (0.5, 0.125) (c) (1, -0.25)

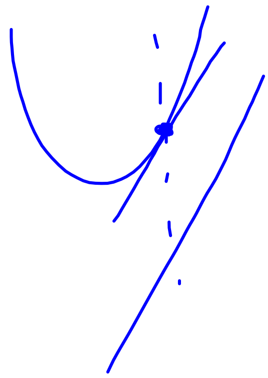
(d) (1, 0.5) (e) (2, 2)

$y' = \frac{1}{2} \cdot 2x$ — der to get slope

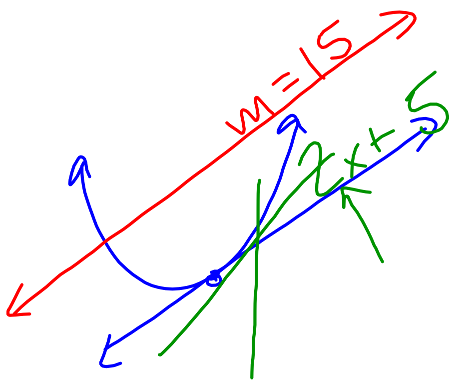
find slope

$y' = x$

$x = \frac{1}{2}$



$y = x^2 + 5x$ $y = 15x - 7$



$15 = 2x + 5$

$x = 5$

3.4b

42. $f(x) = x^2 - \frac{2}{x} + 4$

8.

29.

34.

8.

$Q(t) = 200(30-t)^2$

$= 200(900 - 60t + t^2)$

$= 180{,}000 - 12000t + 200t^2$

$Q'(t) = -12{,}000 + 400t$

$= -8{,}000 \frac{\text{gal}}{\text{min.}}$

$(0, 180{,}000) \qquad (10, 80{,}000)$

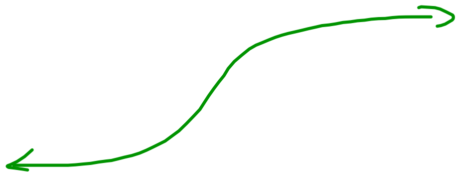
Avg. $= \frac{80{,}000 - 180{,}000}{10 - 0}$

$= -10{,}000 \frac{\text{gal}}{\text{min.}}$

29.

$$P(x) = \frac{10}{1 + 50 \cdot 2^{5-.1x}}$$

a.



b. $x \geq 0$

34.

$$V = \frac{4}{3}\pi r^3 \Big|_{r=2} = \frac{32}{3}\pi \text{ ft}^3$$

$$\frac{4}{3}\pi (2.2)^3 = 14.1975\pi \text{ ft}^3$$

$$\frac{dV}{dr} = 4\pi r^2 \Big|_{r=2} = 16\pi \frac{\text{ft}^3}{\text{ft}}$$

3.5 Derivatives of Trig. Functions

Assuming x is measured in radians, then:

$$\frac{d}{dx}\sin x = \cos x \qquad \frac{d}{dx}\cos x = -\sin x$$

$$\frac{d}{dx}\tan x = \sec^2 x$$

$$\frac{d}{dx}\left(\frac{\sin x}{\cos x}\right) = \frac{\cos x(\cos x) - \sin x(-\sin x)}{(\cos x)^2}$$

$$\frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$$

$$\sec^2 x$$

$$\frac{d}{dx}\cot x = -\csc^2 x$$

$$\frac{d}{dx}\left(\frac{1}{\tan x}\right) = \frac{\tan x(0) - 1(\sec^2 x)}{\tan^2 x}$$

$$\frac{-1}{\cos^2 x} \cdot \frac{\cos^2 x}{\sin^2 x} = \frac{-1}{\sin^2 x}$$

$$-\csc^2 x$$

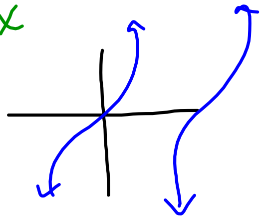
$$\frac{d}{dx}\sec x = \sec x \tan x$$

$$\frac{d}{dx}\left(\frac{1}{\cos x}\right) = \frac{\cos x(0) - 1(-\sin x)}{\cos^2 x}$$

$$\frac{\sin x}{\cos^2 x} = \frac{1}{\cos x} \cdot \frac{\sin x}{\cos x}$$

$$\frac{d}{dx}\csc x = -\csc x \cot x \qquad \sec x \tan x$$

$\sin x$	$\cos x$	$\cos x$	$-\sin x$
$\frac{\sin x}{\cos x}$ $\tan x$	$\sec^2 x$ $\frac{1}{\cos x}$	$\cot x$	$-\csc^2 x$
$\sec x$	$\sec x \tan x$	$\csc x$	$-\csc x \cot x$



Find the derivative of $y = \underbrace{x^2}_{1} \underbrace{\sin x}_{2}$

$$y' = x^2 \cos x + \sin x (2x)$$

$$= x^2 \cos x + 2x \sin x$$

find y'' for $y = \sec x$

$$y' = \sec x \tan x$$

$$y'' = \sec x(\sec^2 x) + \tan x(\sec x \tan x)$$

$$= \sec^3 x + \sec x \tan^2 x$$

Find the eq. of the tangent and normal line of $y = \frac{\tan x}{x}$ at $x = \frac{\pi}{4}$

pt. $\left(\frac{\pi}{4}, \frac{4}{\pi}\right)$ $\quad y = \frac{\tan\frac{\pi}{4}}{\frac{\pi}{4}} = \frac{4}{\pi}$

slope

$$y' = \frac{x(\sec^2 x) - \tan x}{x^2}\Bigg|_{x=\frac{\pi}{4}}$$

$\cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$

$$= \frac{\frac{\pi}{4}\left(\sec^2\frac{\pi}{4}\right) - \tan\frac{\pi}{4}}{\left(\frac{\pi}{4}\right)^2}$$

$$\frac{\frac{\pi}{4}\left(\frac{2}{\sqrt{2}}\right)^2 - 1}{\left(\frac{\pi}{4}\right)^2}$$

$$\frac{\frac{\pi}{2} - \frac{2}{2}}{\left(\frac{\pi}{4}\right)^2} = \frac{\pi - 2}{2} \cdot \left(\frac{4}{\pi}\right)^2$$

$$\text{slope} = \frac{8(\pi-2)}{\pi^2}$$

tangent

$$y = \frac{8(\pi-2)}{\pi^2}\left(x - \frac{\pi}{4}\right) + \frac{4}{\pi}$$

Simple Harmonic Motion

position: $y = 7\sin t$

velocity:

acceleration:

jerk:

When is the particle slowing down?

When is the particle moving fastest?