

**Warm-Up**

1997MC AB12

At what point on the graph of  $y = \frac{1}{2}x^2$  is the tangent line parallel to the line  $2x - 4y = 3$ ?

(a) (0.5, -0.5)      (b) (0.5, 0.125)      (c) (1, -0.25)

(d) (1, 0.5)      (e) (2, 2)

*Handwritten notes:*  
 $y' = \frac{1}{2} \cdot 2x$  (circled) → der to get slope  
 find slope  
 $x = \frac{1}{2}$   
 graph of  $y = \frac{1}{2}x^2$  with a tangent line at  $x = \frac{1}{2}$  and a dashed line representing the target slope.

$y = x^2 + 5x$        $y = 15x - 7$

*Handwritten work:*  
 Graph of  $y = x^2 + 5x$  with a tangent line at a point. The slope of the tangent line is labeled  $m = 15$ . The derivative is labeled  $2x + 5$ .  
 $15 = 2x + 5$   
 $x = 5$

3.4b

$$42. f(x) = x^2 - \frac{2}{x} + 4$$

8.

29.

34.

8.

$$\begin{aligned} Q(t) &= 200(30-t)^2 \\ &= 200(900 - 60t + t^2) \\ &= 180,000 - 12,000t + 200t^2 \end{aligned}$$

$$\begin{aligned} Q'(t) &= -12,000 + 400t \\ &= -8,000 \frac{\text{gal}}{\text{min.}} \end{aligned}$$

$$(0, 180,000) \quad (10, 80,000)$$

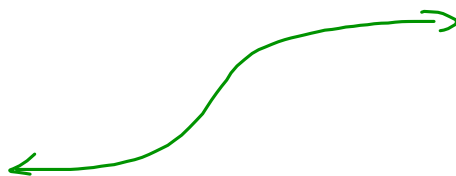
$$\underline{\underline{\text{Avg.}}} = \frac{80,000 - 180,000}{10 - 0}$$

$$= -10,000 \frac{\text{gal}}{\text{min.}}$$

29.

$$P(x) = \frac{10}{1 + 50 \cdot 2^{-5 \cdot 1x}}$$

a.



b.

$$x \geq 0$$

34.

$$V = \frac{4}{3} \pi r^3 \Big|_{r=2} = \frac{32}{3} \pi \text{ ft}^3$$

$$\frac{4}{3} \pi (2.2)^3 = 14.1975 \pi \text{ ft}^3$$

$$\frac{dV}{dr} = 4 \pi r^2 \Big|_{r=2} = 16 \pi \frac{\text{ft}^3}{\text{ft}}$$

### 3.5 Derivatives of Trig. Functions

Assuming  $x$  is measured in radians, then:

$$\frac{d}{dx} \sin x = \cos x \qquad \frac{d}{dx} \cos x = -\sin x$$

$$\frac{d}{dx} \tan x = \sec^2 x$$

$$\frac{d}{dx} \left( \frac{\sin x}{\cos x} \right) = \frac{\cos x (\cos x) - \sin x (-\sin x)}{(\cos x)^2}$$

$$\frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$$

$$\sec^2 x$$

$$\frac{d}{dx} \cot x = -\csc^2 x$$

$$\frac{d}{dx} \left( \frac{1}{\tan x} \right) = \frac{\cancel{\tan x} (0) - 1 (\sec^2 x)}{\tan^2 x}$$

$$\frac{-1}{\cancel{\cos^2 x}} \cdot \frac{\cancel{\cos^2 x}}{\sin^2 x} = \frac{-1}{\sin^2 x}$$

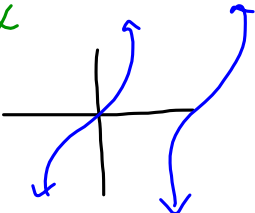
$$-\csc^2 x$$

$$\frac{d}{dx} \sec x = \sec x \tan x$$

$$\frac{d}{dx} \left( \frac{1}{\cos x} \right) = \frac{\cancel{\cos x} (0) - 1 (-\sin x)}{\cos^2 x}$$

$$\frac{\sin x}{\cos^2 x} = \frac{1}{\cos x} \cdot \frac{\sin x}{\cos x}$$

$$\frac{d}{dx} \csc x = -\csc x \cot x \qquad \sec x \tan x$$

$\sin x$	$\cos x$	$\cos x$	$-\sin x$
$\frac{\sin x}{\cos x}$	$\sec^2 x$	$\cot x$	$-\csc^2 x$
$\tan x$	$\frac{1}{\cos x}$		
			
$\sec x$	$\sec x \tan x$	$\csc x$	$-\csc x \cot x$

Find the derivative of  $y = \underbrace{x^2}_1 \underbrace{\sin x}_2$

$$y' = x^2 \cos x + \sin x (2x)$$

$$= x^2 \cos x + 2x \sin x$$

find  $y''$  for  $y = \sec x$

$$y' = \sec x \tan x$$

$$\begin{aligned} y'' &= \sec x (\sec^2 x) + \tan x (\sec x \tan x) \\ &= \sec^3 x + \sec x \tan^2 x \end{aligned}$$

Find the eq. of the tangent and normal line of  $y = \frac{\tan x}{x}$   
at  $x = \frac{\pi}{4}$

pt.  $(\frac{\pi}{4}, \frac{4}{\pi})$   $y = \frac{\tan \frac{\pi}{4}}{\frac{\pi}{4}} = \frac{4}{\pi}$

slope

$$y' = \frac{x(\sec^2 x) - \tan x}{x^2} \Big|_{x = \frac{\pi}{4}}$$

$$\cos \frac{\pi}{4} = \frac{\sqrt{2}}{2}$$

$$= \frac{\frac{\pi}{4} (\sec^2 \frac{\pi}{4}) - \tan \frac{\pi}{4}}{(\frac{\pi}{4})^2}$$

$$\frac{\frac{\pi}{4} (\frac{2}{\sqrt{2}})^2 - 1}{(\frac{\pi}{4})^2}$$

$$\frac{\frac{\pi}{2} - \frac{1}{2}}{(\frac{\pi}{4})^2} = \frac{\pi - 2}{2} \cdot (\frac{4}{\pi})^2$$

$$\text{slope} = \frac{8(\pi - 2)}{\pi^2}$$

tangent

$$y = \frac{8(\pi - 2)}{\pi^2} \left( x - \frac{\pi}{4} \right) + \frac{4}{\pi}$$

## Simple Harmonic Motion

position:  $y = 7 \sin t$

velocity:

acceleration:

jerk:

When is the particle slowing down?

When is the particle moving fastest?