Warm-Up

Let $h(x) = f(x) \cdot g(x)$ and $j(x) = \frac{f(x)}{g(x)}$

$h' = fg' + gf'$ $\quad \frac{gf' - fg'}{g^2}$

Fill in the missing entries in the table below using the information about *f* and *g* given in the table below.

$4 = f(1) + (-2)(2)$ $\quad \frac{-2(2) - 8(1)}{(-2)^2}$ $\quad \frac{-12}{4}$

$8 = f$

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$	$h'(x)$	$j'(x)$
-2	1	-1	-3	4	7	$-\frac{1}{9}$
-1	0	-2	1	1	-2	-2
0	8	2	-2	1	4	-3
1	2		-1	2		-2
2	3	-1		-2		1

3.3.

7. $y = x^3 - 2x^2 + x + 1$

$y' = 3x^2 - 4x + 1 = 0$

$(3x - 1)(x - 1) = 0$

$x = \frac{1}{3}, 1$

12. $y = x^4 - 7x^3 + 2x^2 + 15$

~~14.~~

~~18.~~

41. $y' = 4x^3 - 21x^2 + 4x = 0$

$x(4x^2 - 21x + 4) = 0$

48.

47.

41.

$$y = \frac{4x}{x^2+1}$$

$$y' = \frac{(x^2+1)(4) - 4x(2x)}{(x^2+1)^2}\Bigg|_{x=0}$$

$x = 0$ $(0,0)$

$x = 1$ $(1,2)$

$m = \frac{4}{1}$ $(0,0)$

$y = 4(x-0)+0$

$= 4x$

$\frac{8-8}{1} = 0$

$m = 0$

$y = 0(x-1)+2$

$y = 2$

47.

$$s = 4.9t^2$$

$$\frac{ds}{dt} = 9.8t$$

$$\frac{d^2s}{dt^2} = 9.8$$

48. $R = m^2\left(\frac{c}{2} - \frac{m}{3}\right)$ $-\frac{1}{3}m$

$$\frac{dR}{dm} = m^2\left(-\frac{1}{3}\right) + \left(\frac{c}{2} - \frac{m}{3}\right)2m$$

$$-\frac{1}{3}m^2 + cm - \frac{2}{3}m^2$$

$$\frac{dR}{dm} = cm - m^2$$

14. $y = \frac{x^2+3}{x}$

b. $\frac{x^2}{x} + \frac{3}{x}$

$y = x + 3x^{-1}$

$y' = 1 - 3x^{-2}$

$y' = \frac{x(2x) - (x^2+3)(1)}{x^2}$

$\frac{2x^2 - x^2 - 3}{x^2}$

$\frac{x^2-3}{x^2}$

a. $\frac{x^2-3}{x^2}$

18.

$$y = \frac{x^2+5x-1}{x^2}$$

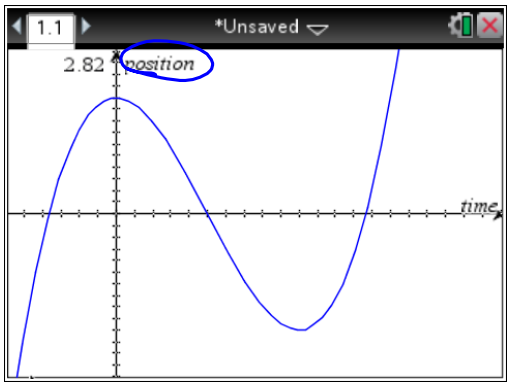
$x^{-2}(x^2+5x-1)$

1. quotient rule 2. product rule

3. $1 + 5x^{-1} - x^{-2}$

3.4a Position, Velocity, Acceleration

How is the position of the particle related to the graph?



(time, distance)

* relate to $t=0$

How is the velocity of the particle related to the graph?

slopes of the position graph

Position $s(t)$ $x(t)$

Velocity ← $s'(t)$ $\frac{ds}{dt}$ $v(t)$

Speed $|v(t)|$ ← velocity w/o direction

Acceleration $s''(t)$ $\frac{d^2s}{dt^2}$ $v'(t)$ $\frac{dv}{dt}$ $a(t)$

Displacement change in position

final − initial

$s(\text{final time}) - s(0)$

At time t = 0, a diver jumps from a platform diving board that is 32 ft. above the water. The position equation is:

$$s(t) = -16t^2 + 16t + 32$$

What is the displacement in the first 2 secs?

$s(2) - s(0) = -16(2)^2 + 16(2) + 32 - (32)$

$= -32$ ft.

Average velocity in the first 2 secs?

$(0, 32)$ $(2, 0)$

$\text{avg. vel.} = \frac{0-32}{2-0} = -16 \frac{\text{ft.}}{\text{sec.}}$

What is the velocity at 2 secs?

$s'(t) = v(t) = -32t + 16\Big|_{t=2} = -48 \frac{\text{ft.}}{\text{sec.}}$

What is the acceleration at 2 secs?

$s''(t) = v'(t) = a(t) = -32 \frac{\text{ft.}}{\text{sec}^2}$

The position of a particle that is moving in a straight line is given by the equation where t is measured in seconds and s in meters.

$$s = t^3 - 6t^2 + 9t$$

(a) Find the velocity at time t.

(b) What is the velocity at 2s? At 4s?

(c) When is the particle at rest? $v(t) = 0$

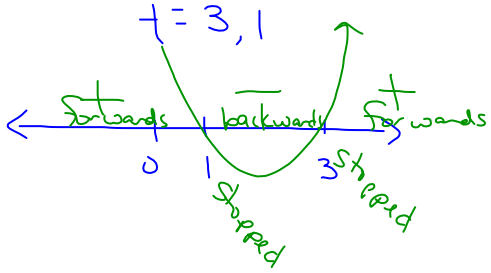
$3t^2 - 12t + 9 = 0$

(d) When is the particle moving forward (that is, in the positive direction)?

$3(t^2 - 4t + 3) = 0$

$3(t-3)(t-1) = 0$

$t = 3, 1$



(e) Describe the motion of the particle.

$[0,1)$ $(3,\infty)$ forward

$(1,3)$ backward

$t = 1, 3$ stopped

(f) Find the displacement of the particle during the first 5 sec.

$s(5) - s(0)$

(g) Find the total distance traveled by the particle in the first 5 sec.

(h) Find the acceleration at time t and at 4s.

(i) Graph the position, velocity, and acceleration functions for

$0 \le t \le 5$